A Blind Channel Estimation Algorithm for Space-Time Coded MC-CDMA Receivers

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***Abstract*—In this paper we propose a linearly constrained minimum variance receiver for space-time coded multicarrier (MC) CDMA system in frequency selective fading channels. It is shown that in the proposed receiver the channel can be blindly estimated as the eigenvector that corresponds to the maximum eigenvalue of an autocorrelation matrix, and then, efficient algorithms for subspace tracking can be used. Computer simulations indicated that the proposed channel estimation algorithm achieves performance comparable to traditional algorithms with less computational cost.**

I. Introduction

An important challenge for fourth generation systems is the selection of an appropriate multiple access scheme which provides the data transmission at rates of 100 Mb/s for high–mobility applications to 1 Gb/s for low–mobility applications [1] and high spectrum efficiency up to 10 b/s/Hz [2]. Two technologies are the keys to meet these requirements. The first one is the use of multicarrier (MC) transmission systems in a multiple access scheme, such as code division multiple access (MC-CDMA). The second technology is the so-called multiple-input multiple-output (MIMO) systems which has gained a lot of attention as an effective diversity technique to combat fading and/or increase the capacity of wireless networks [1], [3]. One of such techniques is the Alamouti space-time coding [4] which uses two transmit and multiple receive antennas.

Blind adaptive linear receivers are promising techniques for interference suppression in CDMA systems, as they offer an attractive trade-off between performance and complexity and can be used in situations where a receiver loses track of the desired signal and a training sequence is not available. A blind adaptive detector for space-time single carrier direct sequence (DS) CDMA systems in flat fading channels was introduced in [5]. It uses a Capon-like structure and requires only the knowledge of the spreading code and timing of the user to perform the detection. In [6] a constrained constant modulus receiver for the frequency selective channel case is proposed.

In this work we propose a constrained minimum variance receiver for space-time multicarrier CDMA system in frequency selective channels that incorporates a new blind channel estimation algorithm. It is shown that the channel can be blindly estimated as the eigenvector that corresponds to the maximum eigenvalue of an autocorrelation matrix, and then, efficient algorithms for subspace tracking can be used. Through computer simulations it is shown that the proposed channel estimation algorithm achieves performance comparable to traditional algorithms with less computational cost.

This paper is structured as follows. Section II describes the space-time MC-CDMA system model. In Section III the proposed constrained minimum variance receiver is introduced and in Section IV a recursive least squares implementation is presented. The new channel estimation algorithm and an efficient implementation is shown in Section V. Some simulation experiments are presented in Section VI, while Section VII gives the conclusions.

II. System Model

A discrete model of a MC-CDMA space-time block coded system employing Alamouti's [4] scheme operating in frequency selective channels is depicted in Fig. 1. This scheme uses two transmit antennas and N receive antennas.

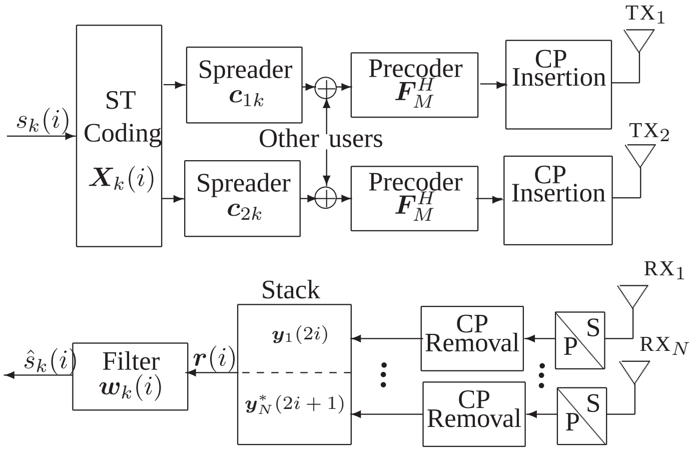

Figure 1. Alamouti MC-CDMA transmission system.

Alamouti MC-CDMA proceeds as follows. The symbols are first space-time coded by the space-time encoder, $X_k(i)$, which maps the symbols of user k according to:

$$X_k(i) = \sqrt{\rho_k} \begin{bmatrix} s_k(2i) & -s_k^*(2i+1) \\ s_k(2i+1) & s_k^*(2i) \end{bmatrix} \qquad (1)$$

where $\rho_k = E_k/2$, E_k is the transmitted energy for user k, and $(\cdot)^*$ denotes complex conjugate. It is assumed that symbols $s_k(i)$, drawn from some constellation with zero mean and unit average symbol energy, are independent and identically distributed (i.i.d.). Different rows of $\boldsymbol{X}_k(i)$ refer to different branches of the transmitter (see Fig. 1) whereas different columns refer to different symbol periods.

Two spreading codes of M binary chips per symbol are assigned to each user, one for each row of $\boldsymbol{X}_k(i)$. Each spread symbol is modulated in multicarrier fashion by the $M \times M$ matrix $\boldsymbol{F}_M$ that implements a M-point DFT, normalized such that, $\boldsymbol{F}_M^H \boldsymbol{F}_M = \boldsymbol{F}_M \boldsymbol{F}_M^H = \boldsymbol{I}_M$. After that, in order to avoid interblock interference (IBI) at the receiver, a cyclic prefix guard interval of length G is inserted, G must be at least the channel order. This operation is represented by a $P \times M$ matrix $\boldsymbol{T}$:

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{0}_{G\times M-G} \,|\, \boldsymbol{I}_G \\ \boldsymbol{I}_M \end{bmatrix}$$

where $P = M + G$, $\boldsymbol{I}_m$ represents a $m \times m$ identity matrix and $\boldsymbol{0}_{m\times n}$ represents an $m \times n$ null matrix. Finally, each block is serialized and transmitted through antenna TX_1 or TX_2, according to $\boldsymbol{X}_k(i)$.

The channel impulse response from the j-th ($j = 1, 2$) transmitter to the n-th ($n = 1, 2, \ldots, N$) antenna in the receiver, $\boldsymbol{h}_{jn}(i)$, is modelled here as a FIR filter with L taps whose gains are samples of the channel impulse response complex envelope.

Assuming that during two symbol periods each multipath channel impulse response remains constant, that is, $\boldsymbol{h}_{jn}(2i) = \boldsymbol{h}_{jn}(2i+1) = [h_{jn,0}(2i) \ldots h_{jn,L-1}(2i)]^T$, the transmission through the multipath channel can be represented by a $P \times P$ lower triangular Toeplitz convolution matrix $\boldsymbol{H}_{jn}(2i)$, whose first column is $[h_{jn,0}(2i) \ \ldots \ h_{jn,L-1}(2i) \ 0 \ \ldots \ 0]^T$.

As we consider a downlink scenario, where the signal for the users experience the same channel conditions, the received vectors collected on the n-th antenna over two consecutive symbol periods are

$$\begin{aligned} \boldsymbol{y}_n(2i) \;&=\; \boldsymbol{H}_{1n}(2i)\boldsymbol{T}\boldsymbol{F}_M^H \sum_{k=1}^{K} \sqrt{\rho_k}\boldsymbol{c}_{1k} s_k(2i) \\ &\quad + \boldsymbol{H}_{2n}(2i)\boldsymbol{T}\boldsymbol{F}_M^H \sum_{k=1}^{K} \sqrt{\rho_k}\boldsymbol{c}_{2k} s_k(2i+1) \\ &\quad + \boldsymbol{n}_n(2i) + \boldsymbol{\eta}_n(2i) \end{aligned} \quad (2)$$

$$\begin{aligned} \boldsymbol{y}_n(2i+1) \;&=\; -\boldsymbol{H}_{1n}(2i)\boldsymbol{T}\boldsymbol{F}_M^H \sum_{k=1}^{K} \sqrt{\rho_k}\boldsymbol{c}_{1k} s_k^*(2i+1) \\ &\quad + \boldsymbol{H}_{2n}(2i)\boldsymbol{T}\boldsymbol{F}_M^H \sum_{k=1}^{K} \sqrt{\rho_k}\boldsymbol{c}_{2k} s_k^*(2i) \\ &\quad + \boldsymbol{n}_n(2i+1) + \boldsymbol{\eta}_n(2i+1) \end{aligned} \quad (3)$$

where $\boldsymbol{n}_n(i)$ is a complex white Gaussian noise vector whose covariance matrix $\mathsf{E}\left[\boldsymbol{n}_n(i)\boldsymbol{n}_n^H(i)\right] = \sigma^2 \boldsymbol{I}_P$ and $\boldsymbol{\eta}_n(i)$ represents the interblock interference (IBI). The operators $(\cdot)^H$ and $\mathsf{E}[\cdot]$ stands for Hermitian transpose and ensemble average, respectively.

The receiver must remove the guard interval from the received signal to eliminate IBI. This removal is represented by the matrix $\boldsymbol{R} = [\boldsymbol{0}_{M\times G} | \ \boldsymbol{I}_M]$. The received column vectors $\boldsymbol{r}_n(2i) = \boldsymbol{R}\boldsymbol{y}_n(2i)$ and $\boldsymbol{r}_n(2i+1) = \boldsymbol{R}\boldsymbol{y}_n^*(2i+1)$ can be rewrite in compact form by noting that

$$\boldsymbol{R}\boldsymbol{H}_{jn}(2i)\boldsymbol{T}\boldsymbol{F}_M^H \boldsymbol{c}_{jk} = \boldsymbol{V}_{jk}\boldsymbol{h}_{jn} \quad (4)$$

where $\boldsymbol{V}_{jk} = \boldsymbol{F}_M^H \mathrm{diag}(\boldsymbol{c}_{jk})\tilde{\boldsymbol{F}}_{M\times L}$, $\mathrm{diag}(\boldsymbol{x})$ is a diagonal matrix with the components of $\boldsymbol{x}$ as its nonzero elements, and $\tilde{\boldsymbol{F}}_{M\times L}$ is a $M \times L$ matrix formed with the first L columns of the matrix that implements the (non-normalized) M-point DFT. Equation (4) comes from the fact that when cyclic prefix is used as guard interval, $\boldsymbol{R}\boldsymbol{H}_{jn}(2i)\boldsymbol{T}$ is a circulant matrix and can be decomposed as $\boldsymbol{F}_M^H \mathrm{diag}(\tilde{\boldsymbol{h}}_{jn}(2i))\boldsymbol{F}_M$, where $\tilde{\boldsymbol{h}}_{jn}(2i)$ is the frequency response of the channel $\boldsymbol{h}_{jn}(2i)$, i.e., $\tilde{\boldsymbol{h}}_{jn}(2i) = \tilde{\boldsymbol{F}}_{M\times L}\boldsymbol{h}_{jn}(2i)$.

Then,

$$\begin{aligned} \boldsymbol{r}_n(2i) \;&=\; \sum_{k=1}^{K} \sqrt{\rho_k}[\boldsymbol{V}_{1k}\boldsymbol{h}_{1n}(2i)s_k(2i) \\ &\quad + \boldsymbol{V}_{2k}\boldsymbol{h}_{2n}(2i)s_k(2i+1)] + \boldsymbol{n}_n(2i) \end{aligned} \quad (5)$$

$$\begin{aligned} \boldsymbol{r}_n(2i+1) \;&=\; \sum_{k=1}^{K} \sqrt{\rho_k}[-\boldsymbol{V}_{1k}^*\boldsymbol{h}_{1n}^*(2i)s_k(2i+1) \\ &\quad + \boldsymbol{V}_{2k}^*\boldsymbol{h}_{2n}^*(2i)s_k(2i)] + \boldsymbol{n}_n^*(2i+1) \end{aligned} \quad (6)$$

Stacking all the column vectors $\boldsymbol{r}_n(2i)$ and $\boldsymbol{r}_n(2i+1)$, $n = 1, \cdots, N$, we get the observation vector

$$\begin{aligned} \boldsymbol{r}(i) \;&=\; \begin{bmatrix} \boldsymbol{r}_1(2i) \\ \boldsymbol{r}_1(2i+1) \\ \vdots \\ \boldsymbol{r}_N(2i) \\ \boldsymbol{r}_N(2i+1) \end{bmatrix} \\ &=\; \sum_{k=1}^{K} \sqrt{\rho_k}\left\{\overline{\boldsymbol{\Psi}}_k \boldsymbol{h}(i)s_k(2i) + \widetilde{\boldsymbol{\Psi}}_k \boldsymbol{h}^*(i)s_k(2i+1)\right\} \\ &\quad + \bar{\boldsymbol{n}}(i) \end{aligned} \quad (7)$$

where

$$\overline{\boldsymbol{\Psi}}_k = \boldsymbol{I}_N \otimes \begin{bmatrix} \boldsymbol{V}_{1k} & \boldsymbol{0}_{M\times L} \\ \boldsymbol{0}_{M\times L} & \boldsymbol{V}_{2k}^* \end{bmatrix} \quad (8)$$

$$\widetilde{\boldsymbol{\Psi}}_k = \boldsymbol{I}_N \otimes \begin{bmatrix} \boldsymbol{0}_{M\times L} & \boldsymbol{V}_{2k} \\ -\boldsymbol{V}_{1k}^* & \boldsymbol{0}_{M\times L} \end{bmatrix} \quad (9)$$

and the $2LN$ dimensional vector

$$\boldsymbol{h}(i) = \begin{bmatrix} \boldsymbol{h}_{11}(2i) \\ \boldsymbol{h}_{21}^*(2i) \\ \vdots \\ \boldsymbol{h}_{1N}(2i) \\ \boldsymbol{h}_{2N}^*(2i) \end{bmatrix}, \quad (10)$$

which we call the composed channel.

III. Linearly Constrained Minimum Variance Receivers

In the following, we assume without lost of generality that user one is desired and drop the user index, k. The design of the receiver filters $\boldsymbol{w} = [\overline{\boldsymbol{w}} \quad \widetilde{\boldsymbol{w}}] \in \mathbb{C}^{2MN\times 2}$, based on the minimum variance (MV) criterion uses the output energy as a cost function to be minimized:

$$\begin{aligned}\boldsymbol{J}_{MV}(\boldsymbol{w}) &= \text{tr}[\boldsymbol{w}^H \boldsymbol{R_{rr}} \boldsymbol{w}] \\ &= \overline{\boldsymbol{w}}^H \boldsymbol{R_{rr}} \overline{\boldsymbol{w}} + \widetilde{\boldsymbol{w}}^H \boldsymbol{R_{rr}} \widetilde{\boldsymbol{w}} \end{aligned} \tag{11}$$

where $\boldsymbol{R_{rr}} = \mathsf{E}\left[\boldsymbol{r}(i)\boldsymbol{r}^H(i)\right]$ and $\text{tr}[\cdot]$ stands for trace.

In order to avoid the trivial solution, $\boldsymbol{w} = \boldsymbol{0}$ and anchor the desired user signal, $\boldsymbol{w}$ is subject to a set of constraints

$$\begin{aligned}\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{w}} = \hat{\boldsymbol{h}} \\ \widetilde{\boldsymbol{\Psi}}^H \widetilde{\boldsymbol{w}} = \hat{\boldsymbol{h}}^*\end{aligned} \tag{12}$$

where $\hat{\boldsymbol{h}}$ is an estimate of the composed channel (10).

Using the method of Lagrange multipliers, the optimum receiver vector is obtained as

$$\begin{aligned}\overline{\boldsymbol{w}}_{opt} = \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}} (\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}})^{-1} \hat{\boldsymbol{h}} \\ \widetilde{\boldsymbol{w}}_{opt} = \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}} (\widetilde{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}})^{-1} \hat{\boldsymbol{h}}^*\end{aligned} \tag{13}$$

and the resulting output variance, given $\hat{\boldsymbol{h}}$, is

$$\begin{aligned}\boldsymbol{J}_{MV}(\boldsymbol{w}_{opt}) = \; & \hat{\boldsymbol{h}}^H (\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}})^{-1} \hat{\boldsymbol{h}} \\ & + \hat{\boldsymbol{h}}^T (\widetilde{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}})^{-1} \hat{\boldsymbol{h}}^*, \end{aligned} \tag{14}$$

The optimization of the channel estimate maximizes (14) as

$$\begin{aligned}\hat{\boldsymbol{h}}_{opt} = \arg \max_{\|\hat{\boldsymbol{h}}\|=1} \; & \left\{ \hat{\boldsymbol{h}}^H \left(\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}}\right)^{-1} \hat{\boldsymbol{h}} \right. \\ & \left. + \hat{\boldsymbol{h}}^T \left(\widetilde{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}}\right)^{-1} \hat{\boldsymbol{h}}^* \right\}. \end{aligned} \tag{15}$$

Using the fact that $\hat{\boldsymbol{h}}^T (\widetilde{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}})^{-1} \hat{\boldsymbol{h}}^*$ is real valued and taking into account the conjugate symmetric properties induced by space-time block codes [5], $\hat{\boldsymbol{h}}_{opt}$ can be estimated simply by

$$\hat{\boldsymbol{h}}_{opt} = \arg \max_{\|\hat{\boldsymbol{h}}\|=1} \hat{\boldsymbol{h}}^H \left(\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}}\right)^{-1} \hat{\boldsymbol{h}}. \tag{16}$$

whose solution is the eigenvector corresponding to the maximum eigenvalue of $\left(\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}}\right)^{-1}$ or to the minimum eigenvalue of $\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}}$. Note that since $\hat{\boldsymbol{h}}_{opt}$ is an eigenvector of $(\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}})^{-1}$, the minimum variance receiver filters in (13) are given by $\overline{\boldsymbol{w}}_{opt} = \alpha \boldsymbol{R_{rr}}^{-1} \overline{\boldsymbol{\Psi}} \hat{\boldsymbol{h}}_{opt}$, $\widetilde{\boldsymbol{w}}_{opt} = \alpha \boldsymbol{R_{rr}}^{-1} \widetilde{\boldsymbol{\Psi}} \hat{\boldsymbol{h}}_{opt}^*$, where α is the eigenvalue associated to the eigenvector $\hat{\boldsymbol{h}}_{opt}$.

In the next sections we consider possible efficient recursive least squares implementations, and derive the new channel estimation algorithm.

IV. A Recursive Least Squares Implementation

The recursive least squares solution in the previous section, uses Kalman RLS recursions to compute recursively $\hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i)$ and the associated matrices and eigenvectors needed for the minimum variance receiver filter parameter vectors. Once $\hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i)$ is updated, we can form the matrix $\overline{\boldsymbol{\Psi}}^H \hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i) \overline{\boldsymbol{\Psi}}$ and compute its minimum eigenvalue and associated eigenvector directly applying SVD decomposition.

The updated formulae are as follows. First compute

$$\hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i) = \frac{1}{\lambda}\left[\hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i-1) - \gamma(i)\boldsymbol{\psi}(i)\boldsymbol{\psi}^H(i-1)\right], \tag{17}$$

where $\boldsymbol{\psi}(i)$ is defined as the Kalman gain vector, $\boldsymbol{\psi}(i) = \hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i-1)\boldsymbol{r}(i)$, $\gamma(i) = \left[\frac{\lambda}{1-\lambda} + \boldsymbol{r}^H(i)\hat{\boldsymbol{R}}_{\boldsymbol{rr}}^{-1}(i-1)\boldsymbol{r}(i)\right]^{-1}$ and $0 < \lambda < 1$ is the forgetting factor.

Then, post-multiplying (17) by $\overline{\boldsymbol{\Psi}}$, $\overline{\boldsymbol{\Gamma}}(i) = \boldsymbol{R_{rr}}^{-1}(i)\overline{\boldsymbol{\Psi}}$ can be updated as [7]:

$$\overline{\boldsymbol{\Gamma}}(i) = \frac{1}{\lambda}\left[\overline{\boldsymbol{\Gamma}}(i-1) - \gamma(i)\boldsymbol{\Psi}(i)\boldsymbol{r}^H(i)\overline{\boldsymbol{\Gamma}}(i-1)\right] \tag{18}$$

and by applying the SVD on $\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)$ we can obtain the channel vector estimate as the eigenvector associated with the minimum eigenvalue of $\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)$. In order to avoid the SVD decomposition, the inverse power method can be used [8]:

$$\begin{aligned}\nu(i) &= \frac{1}{\text{tr}\left[\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)\right]} \\ \hat{\boldsymbol{h}}(i) &= \left[\boldsymbol{I}_{2NL} - \nu(i)\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)\right]\hat{\boldsymbol{h}}(i-1) \\ \hat{\boldsymbol{h}}(i) &= \frac{\hat{\boldsymbol{h}}(i)}{\|\hat{\boldsymbol{h}}(i)\|}\end{aligned} \tag{19}$$

Finally, the receiver vectors are estimated as $\overline{\boldsymbol{w}}(i) = \alpha(i)\overline{\boldsymbol{\Gamma}}(i)\hat{\boldsymbol{h}}(i)$, $\widetilde{\boldsymbol{w}}(i) = \alpha(i)\widetilde{\boldsymbol{\Gamma}}(i)\hat{\boldsymbol{h}}^*(i)$, where $\alpha(i) = \hat{\boldsymbol{h}}^H(i)\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)\hat{\boldsymbol{h}}(i)$.

The value for the input signal autocorrelation matrix at time zero is [7]

$$\hat{\boldsymbol{R}}_{\boldsymbol{rr}}(0) = E_0 \text{diag}(1, \lambda^{-1}, \lambda^{-2}, \cdots, \lambda^{-(M-1)}) \tag{20}$$

where E_0 is the forward prediction error energy and must be a positive value. At instant zero, $\overline{\boldsymbol{\Gamma}}(0) = \boldsymbol{R_{rr}}^{-1}(0)\overline{\boldsymbol{\Psi}}$.

V. A Different Approach: New Channel Estimation Algorithm

If we define the matrices

$$\begin{aligned}\overline{\boldsymbol{Q}} = \boldsymbol{R_{rr}}^{-1}\overline{\boldsymbol{\Psi}}(\overline{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1}\overline{\boldsymbol{\Psi}})^{-1} \\ \widetilde{\boldsymbol{Q}} = \boldsymbol{R_{rr}}^{-1}\widetilde{\boldsymbol{\Psi}}(\widetilde{\boldsymbol{\Psi}}^H \boldsymbol{R_{rr}}^{-1}\widetilde{\boldsymbol{\Psi}})^{-1}\end{aligned} \tag{21}$$

then, the receiver filters in (13) can be rewritten as

$$\begin{aligned}\overline{\boldsymbol{w}}_{opt} &= \overline{\boldsymbol{Q}}\hat{\boldsymbol{h}} \\ \widetilde{\boldsymbol{w}}_{opt} &= \widetilde{\boldsymbol{Q}}\hat{\boldsymbol{h}}^*\end{aligned} \tag{22}$$

and the resulting minimum variance, (11):

$$J_{MV}(\boldsymbol{w}_{opt}) = \hat{\boldsymbol{h}}^H \overline{\boldsymbol{Q}}^H \boldsymbol{R_{rr}} \overline{\boldsymbol{Q}}\hat{\boldsymbol{h}} + \hat{\boldsymbol{h}}^T \widetilde{\boldsymbol{Q}}^H \boldsymbol{R_{rr}} \widetilde{\boldsymbol{Q}}\hat{\boldsymbol{h}}^*. \tag{23}$$

As before, maximizing the minimum variance, and using the fact that $\hat{\boldsymbol{h}}^T \widetilde{\boldsymbol{Q}}^H \boldsymbol{R}_{rr} \widetilde{\boldsymbol{Q}} \hat{\boldsymbol{h}}^*$ is real valued and taking into account the conjugate symmetric properties induced by space-time block codes, $\hat{\boldsymbol{h}}$ can be estimated by

$$\hat{\boldsymbol{h}}_{opt} = \arg \max_{||\hat{\boldsymbol{h}}||=1} \hat{\boldsymbol{h}}^H \left(\overline{\boldsymbol{Q}}^H \boldsymbol{R}_{rr} \overline{\boldsymbol{Q}} \right) \hat{\boldsymbol{h}} \quad (24)$$

Defining $\overline{\boldsymbol{r}}_u(i) = \overline{\boldsymbol{Q}}^H \boldsymbol{r}(i)$, with the autocorrelation matrix $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u} = \mathsf{E}\left[\overline{\boldsymbol{r}}_u(i)\overline{\boldsymbol{r}}_u^H(i)\right] = \overline{\boldsymbol{Q}}^H \boldsymbol{R}_{rr} \overline{\boldsymbol{Q}}$, (24) reduces to

$$\hat{\boldsymbol{h}}_{opt} = \arg \max_{||\hat{\boldsymbol{h}}||=1} \hat{\boldsymbol{h}}^H \overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u} \hat{\boldsymbol{h}}. \quad (25)$$

Thus, the channel vector estimate, $\hat{\boldsymbol{h}}_{opt}$, can be computed by the SVD decomposition, as the eigenvector associated with the maximum eigenvalue of $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}$.

In practice $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}$ is a rank-one updated matrix $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i) = \lambda \overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i-1) + \boldsymbol{r}_u(i)\boldsymbol{r}_u^H(i)$, and therefore, vector $\hat{\boldsymbol{h}}_{opt}$ can be more efficiently computed by subspace tracking algorithms, as will be shown next.

A. Recursive Least Squares Implementation

Using (18), applying the matrix inversion lemma to $\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i)$ and pre-multiplying the result by $\overline{\boldsymbol{\Gamma}}(i)$ we get that $\hat{\overline{\boldsymbol{Q}}}(i) = \overline{\boldsymbol{\Gamma}}(i)(\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(i))^{-1}$ can be estimated recursively as [7]:

$$\overline{\boldsymbol{u}}(i) = \overline{\boldsymbol{\Psi}}^H \boldsymbol{\psi}(i) \quad (26)$$

$$\overline{\boldsymbol{v}}^H(i) = \boldsymbol{r}^H(i) \hat{\overline{\boldsymbol{Q}}}(i-1) \quad (27)$$

$$\hat{\overline{\boldsymbol{Q}}}(i) = \left[\hat{\overline{\boldsymbol{Q}}}(i-1) - \gamma(i)\boldsymbol{\psi}(i)\overline{\boldsymbol{v}}^H(i)\right] \cdot \cdot \left[\boldsymbol{I}_{2NL} + \frac{\overline{\boldsymbol{u}}(i)\overline{\boldsymbol{v}}^H(i)}{\frac{1}{\gamma(i)} - \overline{\boldsymbol{v}}^H(i)\overline{\boldsymbol{u}}(i)}\right], \quad (28)$$

the initialization is performed as $\hat{\overline{\boldsymbol{Q}}}(0) = \overline{\boldsymbol{\Gamma}}(0)(\overline{\boldsymbol{\Psi}}^H \overline{\boldsymbol{\Gamma}}(0))^{-1}$.

To estimate the channel vector without incurring into the high computational cost of the SVD decomposition, we propose the use of two subspace tracking algorithms. The first one is the so-called natural power method [9], which is the fastest of the power methods to compute eigenpairs [10]. The channel estimation algorithm by the natural power method is summarized in Tab. I, where $m(\hat{\boldsymbol{h}}(i))$ denotes the component of maximal magnitude of $\hat{\boldsymbol{h}}(i)$.

Table I
CHANNEL ESTIMATION BY THE NATURAL POWER METHOD

a) Update $\hat{\overline{\boldsymbol{Q}}}(i)$ as in (26)-(28)
b) Compute $\overline{\boldsymbol{r}}_u(i) = \hat{\overline{\boldsymbol{Q}}}^H(i)\boldsymbol{r}(i)$
c) Update $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i) = \lambda \overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i) + \overline{\boldsymbol{r}}_u(i)\overline{\boldsymbol{r}}_u^H(i)$
d) Update the channel estimate using the natural power method algorithm:
$\hat{\boldsymbol{h}}(i) = \overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i)\hat{\boldsymbol{h}}(i-1)$
$\hat{\boldsymbol{h}}(i) = \frac{\hat{\boldsymbol{h}}(i)}{m(\hat{\boldsymbol{h}}(i))}$
d) Compute the filters $\overline{\boldsymbol{w}}(i) = \hat{\overline{\boldsymbol{Q}}}(i)\hat{\boldsymbol{h}}(i)$ and $\widetilde{\boldsymbol{w}}(i) = \hat{\widetilde{\boldsymbol{Q}}}(i)\hat{\boldsymbol{h}}^*(i)$

The second method we propose to use is the PASTd algorithm [11]. This is due to the fact that the autocorrelation matrix $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}(i)$ is rank-one updated and low computational complexity algorithms are available to use. Moreover, it tracks the principal components sequentially and is ideally suited when only the largest eigenvalue and the corresponding eigenvector are needed, as in this case. The overall algorithm for channel and receiver estimator is summarized in Tab. II. Note in Tab. II that the estimation of $\overline{\boldsymbol{R}}_{\boldsymbol{r}_u \boldsymbol{r}_u}$ is not performed, thus reducing the computational cost.

Table II
CHANNEL ESTIMATION BY THE PASTD ALGORITHM

a) Update $\hat{\overline{\boldsymbol{Q}}}(i)$ as in (26)-(28)
b) Compute $\overline{\boldsymbol{r}}_u(i) = \hat{\overline{\boldsymbol{Q}}}^H(i)\boldsymbol{r}(i)$
c) Update the channel estimate using the PASTd algorithm:
$\beta(i) = \hat{\boldsymbol{h}}^H(i-1)\overline{\boldsymbol{r}}_u(i)$
$\alpha(i) = \lambda\alpha(i-1) + \|\beta(i)\|^2$
$\hat{\boldsymbol{h}}(i) = \hat{\boldsymbol{h}}(i-1) + (\overline{\boldsymbol{r}}_u(i) - \hat{\boldsymbol{h}}(i-1)\beta(i))\beta^*(i)/\alpha(i)$
d) Compute the filters $\overline{\boldsymbol{w}}(i) = \hat{\overline{\boldsymbol{Q}}}(i)\hat{\boldsymbol{h}}(i)$ and $\widetilde{\boldsymbol{w}}(i) = \hat{\widetilde{\boldsymbol{Q}}}(i)\hat{\boldsymbol{h}}^*(i)$

1) Remark: The computational cost associated with the estimation of $\overline{\boldsymbol{Q}}(i)$ in (28) is of the same order than the computational cost associated to compute $\overline{\boldsymbol{\Psi}}^H \boldsymbol{\Gamma}(i)$ in (18). However, the computational cost associated to the channel estimation, with dimension $2LN$, by SVD is cubic and by the inverse power method in (19) is quadratic while the cost of estimate the channel by the PASTd algorithm is linear, thus, the proposed algorithm reduces the overall number of operations.

2) Remark: As can be noted, the channel estimates in (16) and (25) are mathematically equivalent. However, as shown numerically in the Section VI, the estimate of $\overline{\boldsymbol{Q}}(i)$ in (28) converges faster than the estimate of $\overline{\boldsymbol{\Gamma}}(i)$ in (18), so the slower convergence of the PASTd algorithm is compensated and the performance, in terms of bit error rate, of the minimum variance receiver using (13) or using (22) for the proposed method, is slightly better for the proposed algorithm.

VI. SIMULATION RESULTS

In this section we present results for a BPSK synchronous Alamouti MC-CDMA system that employ Hadamard sequences of length $M = 16$. In the first experiment we consider a dynamic scenario where the system has initially 3 users, the power level distribution amongst the interferers follow a log-normal distribution with associated standard deviation of 3 dB. After 1000 symbols, 3 additional users enter the system and the power level distribution amongst interferes is loosen with associated standard deviation being increased to 6 dB. Because we focus on a downlink scenario the users experience the same channel conditions. The channel between each antenna in the transmitter and each antenna in the receiver has $L = 4$ paths, whose gains are randomly drawn from a zero-mean complex Gaussian random variable and kept fixed throughout each simulation run. The relative power of each path was set to 0, -3, -6 end -9 dB. The forgetting factor was set to $\lambda = 0.995$ and the initial condition $\hat{\boldsymbol{h}}(0) = [1 \cdots 0]^T$ for the channel estimate was used. A guard interval length of $G = 3$

was assumed. The results are an average of 500 runs. The phase ambiguity derived from the blind channel estimation procedure is eliminated in our simulations by using the phase of the first component of $\boldsymbol{h}$ as a reference.

In Fig. 2 we plot the mean squared error (MSE) of the estimate of $\overline{\boldsymbol{Q}}(i)$ and the estimate of $\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}(i)$, where the mean squared error (MSE) is defined here as

$$
\begin{aligned}
MSE(\overline{\boldsymbol{Q}}(i)) &= \mathsf{E}\left[\|\overline{\boldsymbol{Q}} - \hat{\overline{\boldsymbol{Q}}}(i)\|_F^2\right] \\
MSE(\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}(i)) &= \mathsf{E}\left[\|\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}} - \overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}(i)\|_F^2\right]
\end{aligned}
$$

where $\overline{\boldsymbol{Q}}$ and $\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}$ are analytical matrices and $\|\cdot\|_F$ is the frobenius norm. We plot the MSE for a signal to noise ratio of 15 dB respect to the desired user power level. One antenna ($N = 1$) was used in the receiver. As stated before, the estimate of $\overline{\boldsymbol{Q}}(i)$ converges faster than the estimate of $\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}(i)$.

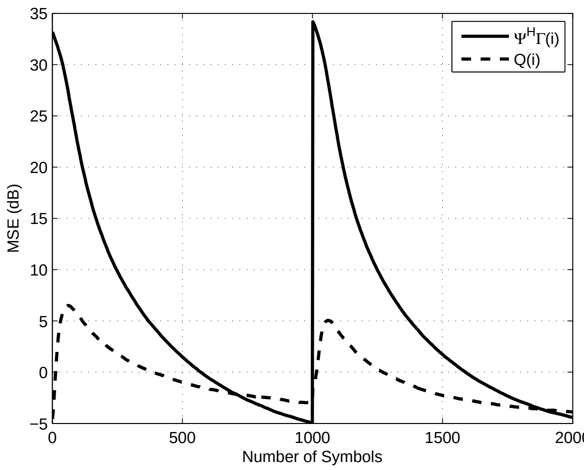

Figure 2. $MSE(\overline{\boldsymbol{Q}}(i))$ and $MSE(\overline{\boldsymbol{\Psi}}^H\overline{\boldsymbol{\Gamma}}(i)), N = 1$

In the same scenario, we assess the performance of the proposed channel estimation in terms of mean squared error (MSE). We compare the channel estimation in (16) by directly applying the SVD decomposition (SVD), the inverse power method in (19) (IPM) and the channel estimation by the PASTd algorithm (PASTd). The channel estimation result using the natural power method (NPM) is not shown because it yielded results similar to the SVD algorithm (with less computational complexity). It is important to stress, however, that the NPM presents higher computational complexity than the PASTd algorithm.

In Fig. 3 we plot the channel estimate mean square error, for signal to noise ratio of 0 dB and 15 dB respect to the desired user power level. One antenna ($N = 1$) was used in the receiver. As it can be observed, the PASTd channel estimate presents a convergence rate comparable to the inverse power method and the SVD, however the minimum error is higher. Similar results are shown in Fig. 4, where two antennas ($N = 2$) were used in the receiver.

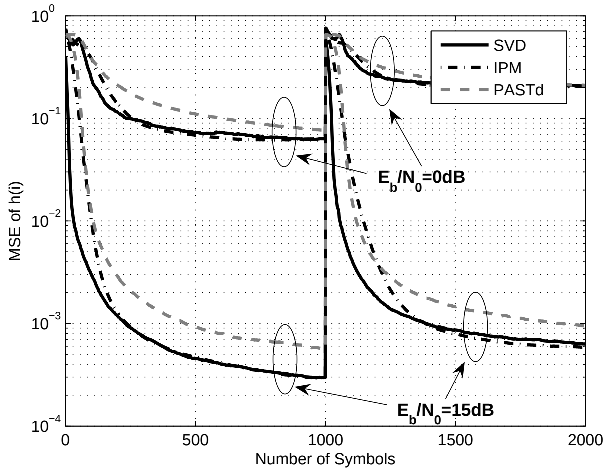

Figure 3. Channel estimation mean square error for dynamic scenario, $N = 1$.

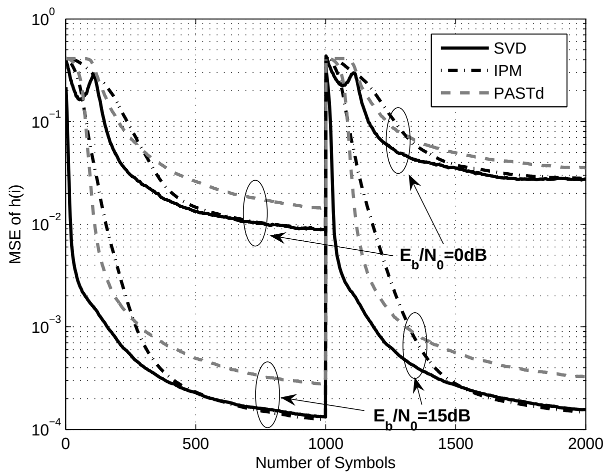

Figure 4. Channel estimation mean square error for dynamic scenario, $N = 2$.

For the next experiment we use a sequence of time-varying channel coefficients for each user, $h_l(i) = p_l\alpha_l(i)$ ($l = 0, 1, 2, \ \ldots, L-1$) obtained with Clarke's model [12]. This procedure corresponds to the generation of L independent sequences of time correlated unit power complex Gaussian random variables ($\mathsf{E}\left[|\alpha_l^2(i)|\right] = 1$) with the path weights p_l normalized so that $\sum_{l=1}^{L_p}|p_l|^2 = 1$. We use a four-path channel ($L = 4$) with relative power of 0, -3, -6 and -9 dB. The channel coefficients change each two-symbol period. The results are shown in terms of the normalized Doppler frequency (f_dT), where f_d is the Doppler frequency and T is the inverse of the symbol rate. In the simulations a $f_dT = 0.0001$ was assumed. The system is loaded with 4 users in a severe near-far scenario where each interferer has a power level 20 dB above the desired user, that is, near-far ratio (NFR) is equal to 20 dB. In Fig. 5the bit error rate (BER) for the three algorithms. One antenna ($N = 1$) was used in the receiver is plotted. It is shown that the proposed method performs as good as the SVD method, while the inverse power method presents an instability for high signal to noise ratios in a time-varying channel. The results are an average of 500 runs, each one consisting of 2000 transmitted symbols.

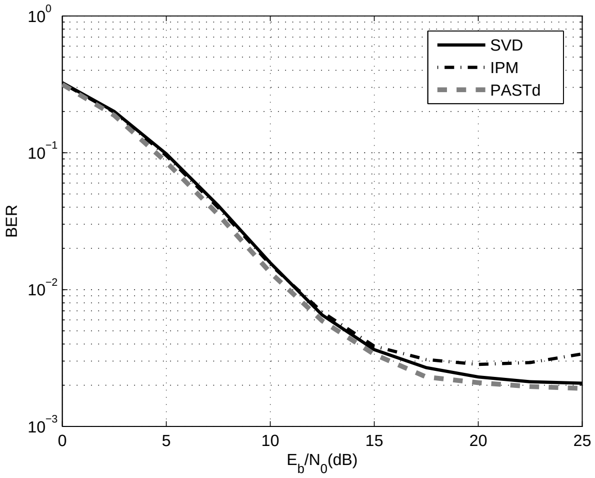

Figure 5. BER for time-varing channel, $f_d T = 0.0001, N = 1$.

VII. Conclusions

In this paper we proposed linearly constrained minimum variance receiver for space-time multicarrier CDMA systems in frequency selective fading channels. A recursive least squares implementation was presented and its was shown that the channel can be blindly estimated as the eigenvector that corresponds to the maximum eigenvalue of an autocorrelation matrix. Efficient algorithms for subspace tracking were used to estimate the channel and it was shown through computer simulations that the proposed channel estimation algorithm achieves performance comparable to traditional algorithms with less computational cost.

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