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POWER TECHNIQUES FOR BLIND CHANNEL ESTIMATION IN ZERO-PADDED OFDM SYSTEMS

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ABSTRACT

This paper revisits the subspace-based channel estimation problem in zero-padded systems and proposes an alternative formulation that leads to a solution to which power techniques are readily applicable. Additionally, a lower-complexity post-DFT implementation that also allows the use of power techniques is derived. Both solutions are shown to be equivalent. Furthermore, links to previous related work are established. Mean square error and bit error rate performance analysis of the estimators through computer simulations reveals that for moderate signal to noise ratio, power techniques with powers as low as $m = 3$ achieves performance similar to the conventional SVD-based estimator.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier block transmission technique that enjoys high spectral efficiency and is robust to frequency selective channels [1]. In order to mitigate interblock interference (IBI), a guard interval is inserted between OFDM blocks. Zero padding (ZP) and cyclic prefix (CP) types of guard intervals are the most common in the literature [2]. One advantage of ZP-OFDM receiver is that it allows full detectability regardless of the location of channel zeros.

Coherent detection entails channel estimation and tracking. One way to perform channel estimation is to multiplex known symbols (pilots) into the data. Pilot-aided channel estimation in multicarrier systems can be performed by inserting pilot symbols on several sub-carriers in the frequency dimension in addition to the time dimension. This approach, however, reduces the system throughput. Blind channel estimation, on the other hand, offers an attractive trade-off between performance, complexity and the need for side information. The need for a training sequence is replaced by some knowledge of the statistical characteristics of the received signal. Subspace-based blind channel estimators have been reported in [3] for CP-OFDM and in [2] for ZP-OFDM. Both works rely on the orthogonality condition between the signal and noise subspaces.

The aim of this work is twofold: the subspace-based channel estimation problem in ZP-OFDM is revisited in order to devise estimators based on power techniques [4, 5]. Also, a lower-complexity implementation of subspace-based estimation methods is derived.

This paper is organized as follows: Section II describes the ZP-OFDM signal model. In section III the blind channel estimation procedure based on subspace and power methods is presented. Section IV presents a lower complexity implementation of the proposed channel estimator. Section V presents the results obtained through computer simulation and, finally, section VI gives the conclusions.

Notation: In what follows, $\mathbf{I}_k$ represents a $k \times k$ identity matrix, $\mathbf{0}_{m\times n}$, an $m \times n$ null matrix, $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, Hermitian transpose and complex conjugate, respectively, $\text{vec}(\mathbf{A})$ is the vector obtained by stacking the columns of $\mathbf{A}$ on top of one another, $\|\mathbf{A}\|_F$ is the Frobenius norm of $\mathbf{A}$ defined as $\left[\text{vec}^H(\mathbf{A})\text{vec}(\mathbf{A})\right]^{\frac{1}{2}}$, $diag(\mathbf{v})$ is a diagonal matrix with the elements of vector $\mathbf{v}$ in its diagonal, $\odot$ represents the Hadamard (element-wise) product, $\star$ is the linear convolution operator, and the operator $\mathbb{E}[\cdot]$ stands for ensemble average.

II. SYSTEM MODEL

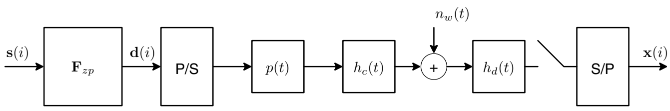

Figure 1: ZP-OFDM System Model

We consider the baseband block equivalent model of a ZP-OFDM system with M subcarriers. The i-th information block $\mathbf{s}(i)$ is precoded by the $P \times M$ matrix $\mathbf{F}_{zp} = [\mathbf{F}_M \;\; \mathbf{0}_{M\times D}]^H$, where $P = M + D$ and $\mathbf{F}_M$ is an $M \times M$ matrix that implements the M-point DFT, normalized such that, $\mathbf{F}_M^H\mathbf{F}_M = \mathbf{F}_M\mathbf{F}_M^H = \mathbf{I}_M$. Thus, the precoder performs an M-point IDFT and inserts a ZP guard interval of length D[1], yielding the time-domain block $\mathbf{d}(i)$.

The resulting time-domain samples of the ZP-OFDM block are then pulse-shaped by $p(t)$ and transmitted through the unknown channel $h_c(t)$. At the receiver end the signal, corrupted by white Gaussian noise $n_w(t)$, is filtered by $h_d(t)$. Denoting $h(t) = p(t) \star h_c(t) \star h_d(t)$, the equivalent baseband channel impulse response, sampling the received signal at P times the OFDM block rate and collecting P samples, we arrive at the following discrete-time vector signal model [2]:

$$\mathbf{x}(i) = \mathbf{H}\mathbf{F}_M^H\mathbf{s}(i) + \mathbf{n}(i), \tag{1}$$

where $\mathbf{s}(i)$ is the i-th transmitted information block, with $\mathbb{E}\left[\mathbf{s}(i)\mathbf{s}^H(i)\right] = \sigma_s^2\mathbf{I}_M$, $\mathbf{H}$ is a $P \times M$ Toeplitz convolution matrix, whose first column is $[\mathbf{h}^T \;\; \mathbf{0}_{1\times P-L-1}]^T$, and first row is $[h_0 \;\, 0 \ldots \;\, 0]$, where $\mathbf{h} = [h_0 \, h_1 \, \ldots \, h_L]^T$ is the discrete-time equivalent channel impulse response of order L (if the channel order is not known *a priori*, it is customary to use D as an upper-bound for it). The complex vector

[1]The guard interval length must be at least the discrete-time equivalent channel order to avoid IBI.

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$\mathbf{n}(i) = [n_0(i) \ \dots \ n_{P-1}(i)]^T$ in (1) contains the samples of the filtered white Gaussian noise and its covariance matrix is $\mathbb{E}\left[\mathbf{n}(i)\mathbf{n}^H(i)\right] = \sigma^2 \mathbf{I}_P$.

III. Blind Channel Estimation

A. *Proposed Method*

Let $\mathbf{R_x}$ be the $P \times P$ autocorrelation matrix of the observation vector $\mathbf{x}(i)$ defined as:

$$\mathbf{R_x} = \mathbb{E}\big[\mathbf{x}(i)\mathbf{x}^H(i)\big]. \tag{2}$$

The Singular Value Decomposition (SVD) of $\mathbf{R_x}$ is given by:

$$\mathbf{R_x} = \big[\mathbf{U}_s \, \mathbf{U}_n\big] \begin{bmatrix} \mathbf{\Lambda}_s + \sigma^2 \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I}_D \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}, \tag{3}$$

where $\mathbf{U}_s$ is a $P \times M$ matrix whose columns form an orthonormal basis for the signal subspace, and $\mathbf{U}_n$ is a $P \times D$ matrix whose columns form an orthonormal basis for the noise subspace. The $M \times M$ matrix $\mathbf{\Lambda}_s$ is a diagonal matrix containing the M singular values associated to the M singular vectors of $\mathbf{U}_s$.

It is well known that the noise subspace is the orthogonal complement of the signal subspace, that is:

$$\mathbf{U}_n^H \mathbf{U}_s = \mathbf{0}_{D \times M}.$$

Therefore, if $\mathbf{a}$ is a vector lying in the signal subspace:

$$\mathbf{U}_n^H \mathbf{a} = \mathbf{0}. \tag{4}$$

We also have from (1) that the columns of $\mathbf{H}$ span the signal subspace (and as a consequence, they lie in it), and therefore the following equalities apply:

$$\mathbf{U}_n^H \mathbf{H} = \mathbf{0}_{D \times M} \quad \Leftrightarrow \quad \|\mathbf{U}_n \mathbf{H}\|_F^2 = 0. \tag{5}$$

Noticing that, due to its Toeplitz structure (see Section II), matrix $\mathbf{H}$ depends only on vector $\mathbf{h}$ (i.e. columns of $\mathbf{H}$ are zero-padded shifted versions of $\mathbf{h}$), it can be rewritten as:

$$\mathbf{H} = \big[\mathbf{S}_1 \mathbf{h} \, \mathbf{S}_2 \mathbf{h} \, \dots \, \mathbf{S}_M \mathbf{h}\big] \text{ with } \mathbf{S}_i = \begin{bmatrix} \mathbf{0}_{(i-1)\times(L+1)} \\ \mathbf{I}_{L+1} \\ \mathbf{0}_{(P-i-L)\times(L+1)} \end{bmatrix},$$

so that: $\|\mathbf{U}_n \mathbf{H}\|_F^2 = \sum_{i=1}^M \|\mathbf{U}_n^H \mathbf{S}_i \mathbf{h}\|^2$.

The orthogonality condition in (5) can therefore be expressed as:

$$\mathbf{U}_n^H \mathbf{H} = \mathbf{0}_{D \times M} \quad \Leftrightarrow \quad \sum_{i=1}^M \|\mathbf{U}_n^H \mathbf{S}_i \mathbf{h}\|^2 = 0 \Leftrightarrow$$
$$\Leftrightarrow \ \mathbf{h}^H \big(\sum_{i=1}^M \mathbf{S}_i^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_i \big) \mathbf{h} = 0, \tag{6}$$

and the channel estimate $\hat{\mathbf{h}}$ of $\mathbf{h}$ is the singular vector associated to the null singular value of $\sum_{i=1}^M \mathbf{S}_i^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_i$ (we constrain $\|\mathbf{h}\| = 1$ to avoid the trivial solution). In fact, $\mathbf{h}$ is estimated up to a complex scalar ambiguity α, for if $\mathbf{h}_0$ is a solution for (6), so is $\alpha \mathbf{h}_0$.

In practice, only estimates of $\mathbf{R_x}$ (and hence estimates of $\mathbf{U}_n$) are available, and the orthogonality condition in (5) is not guaranteed to hold anymore. Therefore, $\hat{\mathbf{h}}$ is chosen as to minimize the quadratic form in (6) (and happens to be the least squares estimate of $\mathbf{h}$), that is:

$$\hat{\mathbf{h}} = \arg \min_{\substack{\mathbf{h} \\ \|\mathbf{h}\|=1}} \Big\{ \mathbf{h}^H \big(\sum_{i=1}^M \mathbf{S}_i^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_i \big) \mathbf{h} \Big\}. \tag{7}$$

Equivalently, $\hat{\mathbf{h}}$ is the singular vector associated to the smallest singular value of $\sum_{i=1}^M \mathbf{S}_i^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_i$.

The product $\mathbf{U}_n \mathbf{U}_n^H$ in (6) can be computed directly from $\mathbf{U}_n$ which in turn is obtained through SVD of $\mathbf{R}_x$ (see (3)) or alternatively it can be estimated by means of the so-called power technique.

The power technique, derived independently in [4] and [5], states that:

$$\lim_{m \to \infty} \big(\sigma^2 \mathbf{R_x}^{-1}\big)^m = \mathbf{U}_n \mathbf{U}_n^H,$$

which means that we may substitute $\mathbf{U}_n \mathbf{U}_n^H$ for ascending powers of $\mathbf{R}_x^{-1}$ and avoid the computation of an SVD of the $P \times P$ matrix $\mathbf{R}_x$. The channel estimator becomes:

$$\hat{\mathbf{h}} = \arg \min_{\substack{\mathbf{h} \\ \|\mathbf{h}\|=1}} \Big\{ \mathbf{h}^H \big(\sum_{i=1}^M \mathbf{S}_i^H (\mathbf{R_x^{-1}})^m \mathbf{S}_i \big) \mathbf{h} \Big\} \tag{8}$$

We stress that as $m \to \infty$, (8)$\to$ (7) and also that, in practice, channel estimation is performed using estimates $\hat{\mathbf{R}}_\mathbf{x}$ of the correlation matrix in (8).

B. *Links with previous works / Relation to other known schemes*

Power techniques have been successfully used in CDMA systems [4] for channel estimation purposes. For classical subspace-based channel estimation, knowledge of the noise subspace dimension is mandatory to extract $\mathbf{U}_n$ and compute $\mathbf{U}_n \mathbf{U}_n^H$. As in CDMA systems the number of active users is not known *a priori*, it follows that the dimension of the noise subspace is also not known *a priori* and must be estimated. Mismatch between the correct an the estimated dimensions leads to large performance degradation of the channel estimator [4]. In order to bypass this shortcoming, [4] and [5] independently proposed a technique based on the power of the correlation matrix to estimate the product $\mathbf{U}_n \mathbf{U}_n^H$.

For ZP-OFDM systems, the noise subspace dimension is known *a priori* as the number of transmitted subcarriers remains fixed during transmission, and $\mathbf{U}_n$ can be easily extracted from $\mathbf{R}_x$ at the cost of a SVD applied to a large $P \times P$ matrix. This kind of subspace-based estimation channel scheme has already been addressed in [2] and is summarized in the sequel.

For the ZP-OFDM system in (1), the noise subspace has dimension D and is spanned by the columns of $\mathbf{U}_n$:

$$\mathbf{U}_n = [\mathbf{q}_1 \ \mathbf{q}_2 \ \cdots \ \mathbf{q}_D] \tag{9}$$

For every vector in the noise subspace and more particularly for each vector in $\mathbf{U}_n$, the following orthogonality condition holds:

$$\mathbf{q}_i^H \mathbf{H} = \mathbf{h}^T \mathbf{Q}_i^* = \mathbf{0}_{M\times 1}^T, \tag{10}$$

where $\mathbf{Q}_i$ is a $(L+1)\times M$ Hankel matrix shown below:

$$\mathbf{Q} = \begin{bmatrix} q_1 & q_2 & \dots & q_M \\ q_2 & q_3 & \dots & q_{M+1} \\ \vdots & \vdots & \dots & \vdots \\ q_{L+1} & q_{L+2} & \dots & q_{M+L} \end{bmatrix}. \tag{11}$$

Thus, a channel estimate $\hat{\mathbf{h}}$ can be obtained as the solution of:

$$\begin{aligned} \hat{\mathbf{h}} &= \arg \min_{\substack{\mathbf{h} \\ \|\mathbf{h}\|=1}} \left\{ \sum_{i=1}^{D} \|\mathbf{Q}_i^H \mathbf{h}\|^2 \right\} \\ &= \arg \min_{\substack{\mathbf{h} \\ \|\mathbf{h}\|=1}} \left\{ \mathbf{h}^H \big(\sum_{i=1}^{D} \mathbf{Q}_i \mathbf{Q}_i^H \big) \mathbf{h} \right\}, \end{aligned} \tag{12}$$

that is, $\hat{\mathbf{h}}$ is the singular vector associated to the smallest singular value of $\big(\sum_{i=1}^{D} \mathbf{Q}_i \mathbf{Q}_i^H\big)$.

We now show that the quadratic forms in (12) and in (7) are the same, that is:

$$\mathbf{h}^H \big(\sum_{i=1}^{D} \mathbf{Q}_i \mathbf{Q}_i^H \big) \mathbf{h} = \mathbf{h}^H \Big(\sum_{j=1}^{M} \mathbf{S}_j^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_j \Big) \mathbf{h}. \tag{13}$$

From (10):

$$\begin{aligned} \sum_{i=1}^{D} \|\mathbf{Q}_i^H \mathbf{h}\|^2 &= \sum_{i=1}^{D} \|\mathbf{q}_i^H \mathbf{H}\|^2 \\ &= \sum_{i=1}^{D} \sum_{j=1}^{M} |\mathbf{q}_i^H \mathbf{S}_j \mathbf{h}|^2 \\ &= \sum_{i=1}^{D} \sum_{j=1}^{M} \mathbf{h}^H \mathbf{S}_j^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{S}_j \mathbf{h} \\ &= \mathbf{h}^H \left[\sum_{j=1}^{M} \mathbf{S}_j^H \Big(\sum_{i=1}^{D} \mathbf{q}_i \mathbf{q}_i^H \Big) \mathbf{S}_j \right] \mathbf{h} \\ &= \mathbf{h}^H \Big(\sum_{j=1}^{M} \mathbf{S}_j^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_j \Big) \mathbf{h}, \end{aligned}$$

where in the last equation we used the fact that $\sum_{i=1}^{D} \mathbf{q}_i \mathbf{q}_i^H = \mathbf{U}_n \mathbf{U}_n^H$. The equality in (13) means that both schemes will lead to the same channel estimate. It should be noticed, however, that in (12), the product $\mathbf{U}_n \mathbf{U}_n^H$ does not appear explicitly and the power techniques are not readily applicable, as opposed to the proposed form in (6).

IV. Alternative Lower Complexity (Post-DFT) Implementation

In this section, we derive a subspace-based channel estimator along the lines of the one proposed in Section III-A, for the post-DFT signal:

$$\mathbf{y}(i) = \mathbf{F}_P \mathbf{x}(i).$$

From [2]:

$$\mathbf{y}(i) = \mathbf{D}(\tilde{\mathbf{h}}) \mathbf{V} \mathbf{s}(i) + \mathbf{n}'(i), \tag{14}$$

where $\mathbf{D}(\tilde{\mathbf{h}}) = diag(\tilde{\mathbf{h}})$ with

$$\begin{aligned} \tilde{\mathbf{h}} &= \sqrt{P} \mathbf{F}_P \begin{bmatrix} \mathbf{h} \\ \mathbf{0}_{(P-L-1)\times 1} \end{bmatrix} \\ &= \sqrt{P} \mathbf{F}_P \begin{bmatrix} \mathbf{I}_{(L+1)} \\ \mathbf{0}_{(P-L-1)\times(L+1)} \end{bmatrix} \mathbf{h} \\ &= \mathbf{F}_{P,L+1} \mathbf{h}, \end{aligned} \tag{15}$$

that is, $\tilde{\mathbf{h}}$ is the P-point-sampled frequency response of channel $\mathbf{h}$, and $\mathbf{F}_{P,L+1} = \sqrt{P} \mathbf{F}_P \begin{bmatrix} \mathbf{I}_{(L+1)} \\ \mathbf{0}_{(P-L-1)\times(L+1)} \end{bmatrix}$. Indeed, $\mathbf{F}_{P,L+1}$ is simply $\mathbf{F}_P$ truncated to its first $L+1$ columns and multiplied by $\sqrt{P}$. Matrix $\mathbf{V}$ in (14) is given by

$$\mathbf{V} = \mathbf{F}_P \begin{bmatrix} \mathbf{F}_M^H \\ \mathbf{0}_{D\times M} \end{bmatrix}. \tag{16}$$

Also in (14), $\mathbf{n}(i)' = \mathbf{F}_P \mathbf{n}(i)$ is a complex white Gaussian noise vector with covariance matrix $\mathbb{E}\left[\mathbf{n}'(i)\mathbf{n}'^H(i)\right] = \sigma^2 \mathbf{I}_P$ (due to the fact that $\mathbf{F}_P$ is unitary).

The $P \times P$ autocorrelation matrix of the observation vector $\mathbf{y}(i)$, $\mathbf{R}_\mathbf{y}$, is given by

$$\mathbf{R}_\mathbf{y} = \mathbb{E}\left[\mathbf{y}(i)\mathbf{y}^H(i)\right], \tag{17}$$

and has the following SVD:

$$\mathbf{R}_\mathbf{y} = \left[\mathbf{U}'_s \; \mathbf{U}'_n\right] \begin{bmatrix} \mathbf{\Lambda}'_s + \sigma^2 \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I}_D \end{bmatrix} \begin{bmatrix} \mathbf{U}'^H_s \\ \mathbf{U}'^H_n \end{bmatrix}. \tag{18}$$

We also have from (14) that the columns of $\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}$ lie in the signal subspace, and therefore the following equalities apply:

$$\mathbf{U}'^H_n \mathbf{D}(\tilde{\mathbf{h}}) \mathbf{V} = \mathbf{0}_{D\times M} \quad \Leftrightarrow \quad \|\mathbf{U}'^H_n \mathbf{D}(\tilde{\mathbf{h}}) \mathbf{V}\|_F^2 = 0. \tag{19}$$

Noticing that $\mathbf{D}(\tilde{\mathbf{h}}) = diag\big([\,\tilde{h}_1 \; \tilde{h}_2 \; \dots \; \tilde{h}_P\,]^T\big)$ and $\mathbf{V} = \left[\mathbf{v}_1 \; \mathbf{v}_2 \; \dots \; \mathbf{v}_M\right]$, we can write:

$$\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V} = \left[diag(\mathbf{v}_1)\tilde{\mathbf{h}} \; diag(\mathbf{v}_2)\tilde{\mathbf{h}} \; \dots \; diag(\mathbf{v}_M)\tilde{\mathbf{h}}\right].$$

Hence,

$$\begin{aligned} \|\mathbf{U}'^H_n \mathbf{D}(\tilde{\mathbf{h}}) \mathbf{V}\|_F^2 &= \sum_{k=1}^{M} \tilde{\mathbf{h}}^H diag(\mathbf{v}_k)^H \mathbf{U}'_n \mathbf{U}'^H_n diag(\mathbf{v}_k) \tilde{\mathbf{h}} \\ &= \tilde{\mathbf{h}}^H \left[\sum_{k=1}^{M} diag(\mathbf{v}_k)^H \mathbf{U}'_n \mathbf{U}'^H_n diag(\mathbf{v}_k) \right] \tilde{\mathbf{h}} \\ &= \tilde{\mathbf{h}}^H \left[\sum_{k=1}^{M} \big(\mathbf{v}_k^* \mathbf{v}_k^T \odot \mathbf{U}'_n \mathbf{U}'^H_n \big) \right] \tilde{\mathbf{h}} \quad (20) \\ &= \tilde{\mathbf{h}}^H \left[\mathbf{V}^* \mathbf{V}^T \odot \mathbf{U}'_n \mathbf{U}'^H_n \right] \tilde{\mathbf{h}} \quad (21) \\ &= \mathbf{h}^H \mathbf{F}_{P,L+1}^H \left[\mathbf{V}^* \mathbf{V}^T \odot \mathbf{U}'_n \mathbf{U}'^H_n \right] \mathbf{F}_{P,L+1} \mathbf{h} \quad (22) \end{aligned}$$

where we used the following facts:

$$diag(\mathbf{u})\mathbf{A}diag(\mathbf{v}) = \left(\mathbf{u}\mathbf{v}^T\right) \odot \mathbf{A} \tag{23}$$

and

$$\sum_i \left(\mathbf{B}_i \odot \mathbf{A}\right) = \left(\sum_i \mathbf{B}_i\right) \odot \mathbf{A}$$

in equations (20) and (21) respectively (for a proof of (23), see Appendix), and (15) in (22). In light of (22), we estimate $\hat{\mathbf{h}}$ as the singular vector associated with the smallest singular value of the smaller $(L+1) \times (L+1)$ matrix $\mathbf{F}_{P,L+1}^H\left[\mathbf{V}^*\mathbf{V}^T \odot \mathbf{U'}_n\mathbf{U'}_n^H\right]\mathbf{F}_{P,L+1}$.

Observing that $\mathbf{U'}_n\mathbf{U'}_n^H$ appears explicitly in (22), power techniques are readily applicable:

$$\hat{\mathbf{h}} = \arg\left\{\min_{\mathbf{h}} \mathbf{F}_{P,L+1}^H\left[\mathbf{V}^*\mathbf{V}^T \odot \mathbf{R}_y^{-m}\right]\mathbf{F}_{P,L+1}\right\}. \tag{24}$$

We now show that although derived independently, the quadratic forms in (22) and (6) (and hence in (12)) are indeed equivalent, that is:

$$\|\mathbf{U}_n^H\mathbf{H}\|_F^2 = \|\mathbf{U'}_n^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\|_F^2.$$

First consider:

$$\mathbf{x}(i) = \mathbf{F}_P^H\mathbf{y}(i) = \mathbf{F}_P^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\mathbf{s}(i) + \mathbf{n}(i). \tag{25}$$

Comparing (25) and (1), we conclude that

$$\mathbf{H} = \mathbf{F}_P^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\mathbf{F}_M.$$

We next notice that $\mathbf{R_y} = \mathbf{F}_P\mathbf{R_x}\mathbf{F}_P^H$ and thus $\mathbf{R_y}$ can be seen as obtained from $\mathbf{R_x}$ by a *similarity transformation* (thanks to $\mathbf{F}_P$ being unitary). As a consequence, in (18), $\mathbf{\Lambda}'_s = \mathbf{\Lambda}_s$ and $\left[\mathbf{U'}_s\ \mathbf{U'}_n\right] = \mathbf{F}_P\left[\mathbf{U}_s\ \mathbf{U}_n\right]$.

Then,

$$\begin{aligned}\|\mathbf{U}_n^H\mathbf{H}\|_F^2 &= \|\mathbf{U}_n^H\mathbf{F}_P^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\mathbf{F}_M^H\|_F^2 \\ &= \|\mathbf{U}_n^H\mathbf{F}_P^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\|_F^2 \\ &= \|\mathbf{U'}_n^H\mathbf{D}(\tilde{\mathbf{h}})\mathbf{V}\|_F^2.\end{aligned}$$

The formulation in (22) leads to a more attractive implementation. In opposition to (12) and (7), (22) has no summation and the matrix products involving $\mathbf{U}_n\mathbf{U}_n^H$ in (7) are replaced with simpler element-wise products. This comes especially in hand in the case when estimates of $\mathbf{U}_n\mathbf{U}_n^H$ are constantly updated, as they can be simply plugged into (22). (Notice that matrix $\mathbf{V}^*\mathbf{V}^T$ in (24) can be pre-computed.)

V. Simulation Results

The simulation results presented are for a zero-padded OFDM system with $M = 64$ subcarriers carrying BPSK-modulated symbols. The communication channel is modelled as an FIR filter with order $L = 11$. A guard interval of length $D = 16$ is inserted to allow for interblock interference suppression at the receiver. The multipath gains are randomly drawn from a zero-mean complex Gaussian random variable and *kept fixed throughout the experiment*. These gains are normalized in order to have $\|\mathbf{h}\|^2 = 1$. The results are an average of 100 simulation runs. The phase ambiguity derived from the blind channel estimation is eliminated in our simulations by using the phase of h_0 as a reference. In the experiments, we compare the proposed power method to the blind channel estimator mentioned in [2] denoted "svd". The "svd" estimator works in two steps. The first step consists in the application of an SVD onto a $P \times P$ observation correlation matrix in order to obtain a base for the noise subspace of the received signal. After that, a second SVD applied to a smaller $(L+1) \times (L+1)$ matrix, yields the channel estimate up to a complex scalar ambiguity. The correlation matrix is estimated using $\hat{\mathbf{R}}_\mathbf{x}(i) = (1/i)\sum_{j=1}^{i}\mathbf{x}(j)\mathbf{x}^H(j)$.

Figs. 2-4 depict mean square error (MSE) performance of both estimators versus number of transmitted blocks for $E_b/N_0 = 10, 15, 20, 25$ dB, respectively. The figures clearly show that as $m \to \infty$, the proposed power method MSE converges to the same result of the "svd" estimator. Also, we notice that for $E_b/N_0 > 15$ dB, the power method, using $m = 3$, achieves performance close to the "svd" estimator.

Fig. 5 depicts bit error rate (BER) performance versus E_b/N_0, for a system transmitting BPSK modulated symbols (with $M = 16$, $D = 4$, $L = 3$) using a zero-forcing equalizer, for the "svd" and several values of m. It can be seen that for a wide range of E_b/N_0 values, a system using power-techniques based estimator with m as low as 3 exhibits performance close to that of a system using the "svd" estimator.

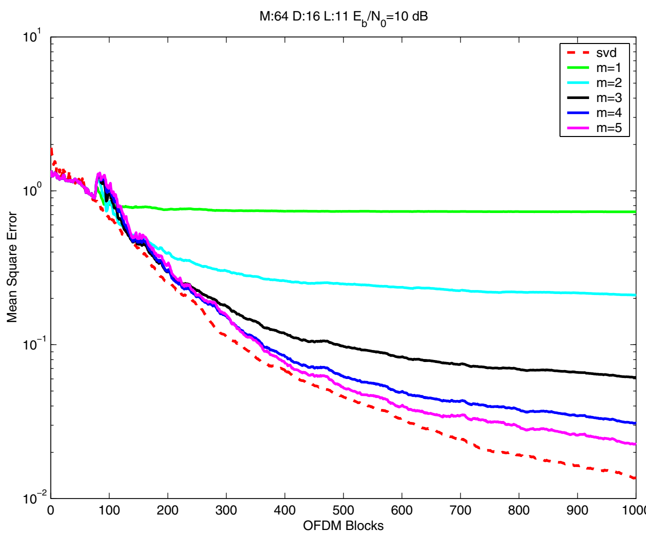

Figure 2: MSE convergence performance, $E_b/N_0 = 10$ dB

VI. Conclusions

This work revisited the subspace-based channel estimation problem for ZP-OFDM systems and proposed an alternative formulation that lead to an lower-complexity post-DFT implementation and allowed the use of power techniques instead of a large SVD. Mean square error and bit error rate performance were assessed and the results showed that for moderate E_b/N_0 the channel estimation based on power techniques with powers

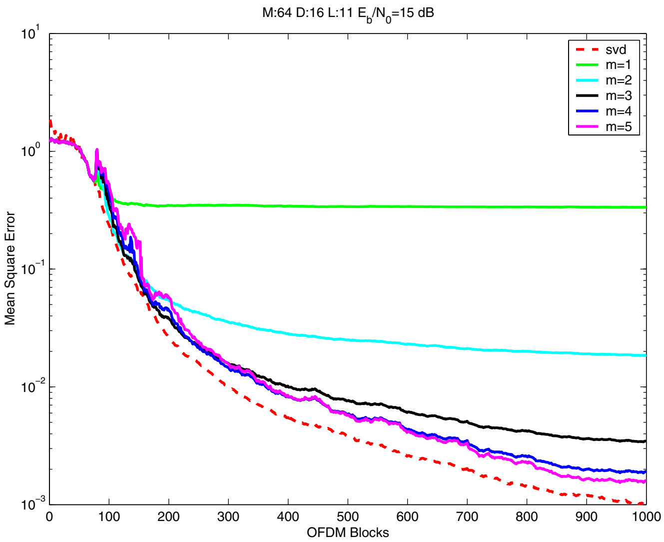

Figure 3: MSE convergence performance, $E_b/N_0 = 15$ dB

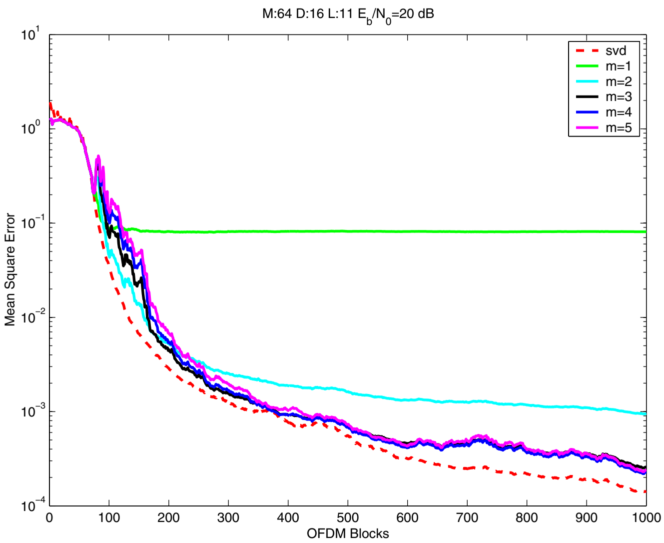

Figure 4: MSE convergence performance, $E_b/N_0 = 20$ dB

as low as $m = 3$ presents performance similar to the SVD-based channel estimator.

Acknowledgement

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A Appendix

In what follows, $M_{p,q}$ denotes the $\{p, q\}$-entry of matrix $\mathbf{M}$, and v_p the p-th element of vector $\mathbf{v}$. We show that:

$$diag(\mathbf{u})\mathbf{A}diag(\mathbf{v}) = \left(\mathbf{u}\mathbf{v}^T\right) \odot \mathbf{A}. \qquad (26)$$

First consider:

$$\mathbf{C} = diag(\mathbf{u})\mathbf{A}diag(\mathbf{v}).$$

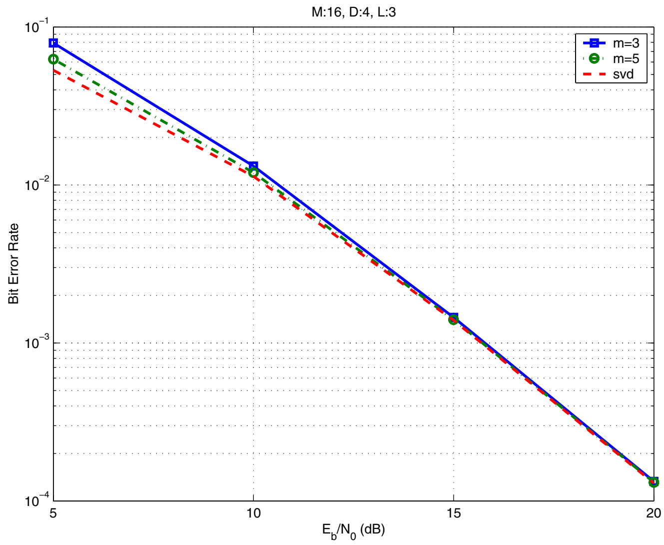

Figure 5: Bit Error Rate Performance

Let

$$\mathbf{B} = \mathbf{A}diag(\mathbf{v})$$

with $B_{p,q} = A_{p,q}v_q$. Then

$$\mathbf{C} = diag(\mathbf{u})\mathbf{B}$$

with $C_{p,q} = u_p A_{p,q} v_q = (u_p v_q)A_{p,q}$.

Now consider:

$$\mathbf{E} = \mathbf{u}\mathbf{v}^T \odot \mathbf{A}.$$

Let

$$\mathbf{D} = \mathbf{u}\mathbf{v}^T$$

with $D_{p,q} = u_p v_q$. It follows that

$$\mathbf{E} = \mathbf{D} \odot \mathbf{A}$$

with $E_{p,q} = (u_p v_q)A_{p,q}$. Hence

$$\mathbf{E} = \mathbf{C},$$

and (26) holds.

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