Low Complexity Blind Constrained Data-Reusing Algorithms Based on Minimum Variance and Constant Modulus Criteria

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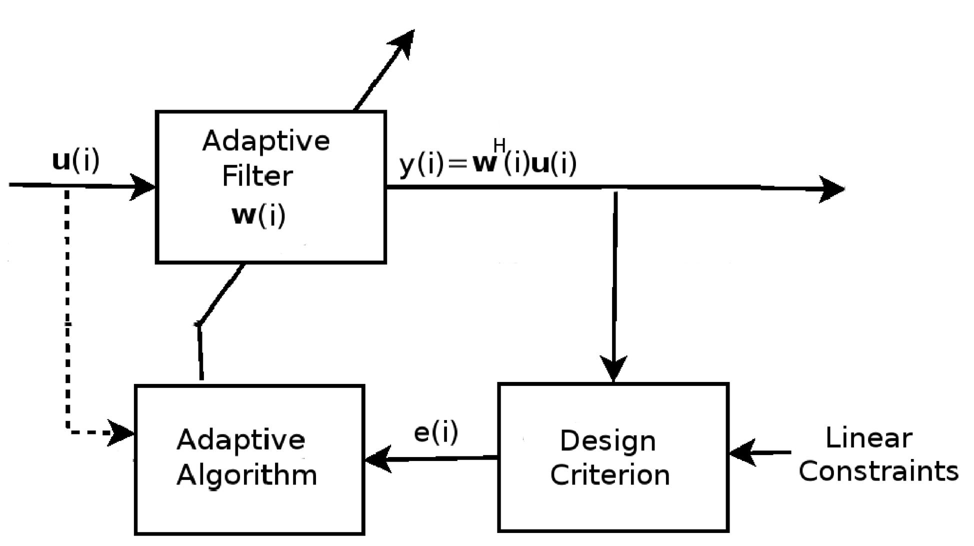
Abstract

This work presents low complexity blind constrained data-reusing adaptive filtering algorithms based on the minimum variance and constant modulus cost functions. Constrained minimum variance (CMV) and constrained constant modulus (CCM) affine projection type algorithms are developed and investigated in a CDMA interference suppression scenario. Computer simulations are used to analyze the proposed techniques and compare them with existing stochastic gradient (SG) and recursive least-squares (RLS) type techniques. The results show that the new algorithms outperform previously reported SG techniques with small additional computational requirements and achieve a performance very close to RLS algorithms at greatly reduced complexity.

Blind Adaptive Filtering and Algorithms

Goal: Obtain an M-dimensional parameter vector $\mathbf{w}$ that minimizes the cost function defined as the design criterion in order to retrieve a desired signal obtained from the $M \times 1$ observation vector $\mathbf{u}$.



- Stochastic Gradient (SG);
  Recursive Least Squares (RLS);
  Affine Projection Algorithm (AP).

Definitions:

- $\mathbf{e}(i)$ is the $P \times 1$ error vector;
  $\mathbf{U}(i) = [\mathbf{u}(i) \ \ldots\ldots \ \mathbf{u}(i-P+1)]$ is a $M \times P$ matrix containing P observation vectors.
- Cost function: Sum of squared errors $\mathbf{e}^H(i)\mathbf{e}(i)$.
  Constraints: $\mathbf{C}^H\mathbf{w} = \mathbf{g}$;
  $\mathbf{C}$ is an $M \times L$ matrix, $M \geq L$ (“tall”).

Constrained Minimum Variance Affine Projection Algorithm

For the MV criterion, the $P \times 1$ error vector $\mathbf{e}(i)$ is

$$\mathbf{e}(i) = \mathbf{U}^H(i)\mathbf{w}(i)$$

Lagrangian:

$$\boxed{\mathcal{L}_{\text{MV}} = \mathbf{w}^H(i)\mathbf{U}(i)\mathbf{U}^H(i)\mathbf{w}(i) + \Re\left[(\mathbf{C}^H\mathbf{w} - \mathbf{g})^H\boldsymbol{\lambda}\right]}$$

Update Recursion:

$$\boxed{\mathbf{w}(i+1) = \boldsymbol{\Pi}\left[\mathbf{w}(i) - \mathbf{U}(i)\boldsymbol{\mu}\mathbf{e}(i)\right] + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}(i)}$$

where $\boldsymbol{\Pi} = \left[\mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\right]$.

Normalized Step-Size:

In order to devise a normalized version of the algorithm, we introduce a convenient $P \times P$ matrix step-size $\boldsymbol{\mu}$.

$$\boxed{\boldsymbol{\mu} = \mu_0\left[\mathbf{U}^H(i)\boldsymbol{\Pi}\mathbf{U}(i)\right]^{-1}}$$

Constrained Constant Modulus Affine Projection Algorithm

For the CM criterion, the jth component of the $P \times 1$ error vector $\mathbf{e}(i)$ is $e_j(i) = |\mathbf{w}^H(i)\mathbf{u}(i-j)|^2 - 1$.

Lagrangian:

$$\boxed{\mathcal{L}_{\text{CM}} = \sum_{j=0}^{P-1}\left[|\mathbf{w}^H(i)\mathbf{u}(i-j)|^2 - 1\right]^2 + \Re\left[(\mathbf{C}^H\mathbf{w} - \nu\mathbf{g})^H\boldsymbol{\lambda}\right]}$$

Update Recursion:

$$\boxed{\mathbf{w}(i+1) = \boldsymbol{\Pi}\left[\mathbf{w}(i) - \mathbf{U}(i)\mathbf{Z}(i)\boldsymbol{\mu}\mathbf{e}(i)\right] + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\nu\mathbf{g}(i)}$$

where $\mathbf{Z}(i) = \text{diag}\left[z_0^*(i), \ldots, z_{P-1}^*(i)\right]$ and $z_j(i) = \mathbf{w}^H(i)\mathbf{u}(i-j)$.

Normalized Step-Size:

$$\boxed{\boldsymbol{\mu} = \mu_0\mathbf{M}\left[\mathbf{U}^H(i)\boldsymbol{\Pi}\mathbf{U}(i)\right]^{-1}}$$

where $\mathbf{M} = \text{diag}\left(\frac{1}{|z_0(i)|(|z_0(i)|-1)}, \ \ldots \ , \ \frac{1}{|z_{P-1}(i)|(|z_{P-1}(i)|-1)}\right)$.

DS-CDMA Interference Suppression

$$\mathbf{u}(i) = \sum_{k=1}^{K} A_k b_k(i)\mathbf{C}_k\mathbf{h}_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i)$$

- Uplink connection of a BPSK DS-CDMA system.
- K users, processing gain N, multipath channel: L_{p} paths.
- $\mathbf{u}(i)$ is a $(N + L_{\text{p}} - 1) \times 1$ vector.
- $\mathbf{h}_k(i) = [h_{k,0}(i) \ \ldots \ h_{k,L_{\text{p}}-1}(i)]^T$ is the channel vector and $\mathbf{C}_k$ contains one-chip shifted versions of the signature sequence for user k, $\boldsymbol{\eta}(i)$ is the ISI and $\mathbf{n}(i)$ is AWGN.

Receiver	Constraints
CMV	$\mathbf{C}_k^H\mathbf{w}_k(i+1) = \mathbf{h}_k(i)$
CCM	$\mathbf{C}_k^H\mathbf{w}_k(i+1) = \nu\mathbf{h}_k(i)$

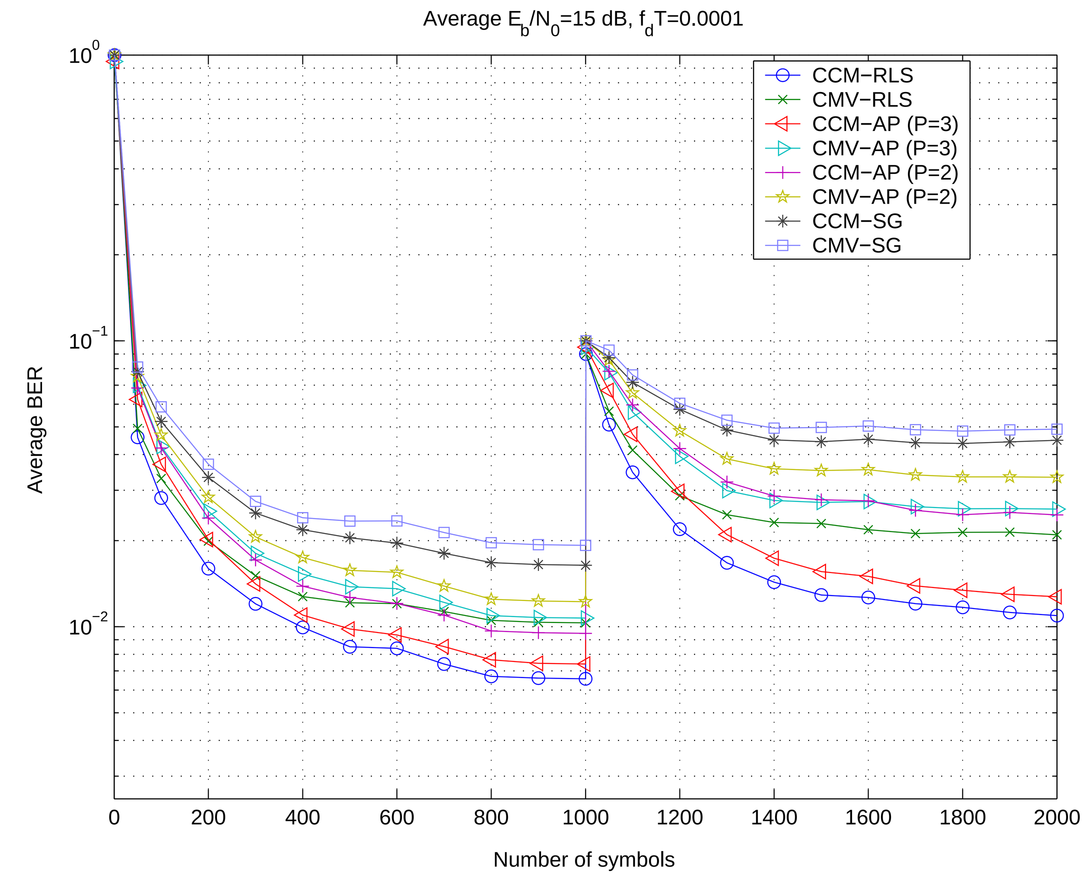
Simulations and Results

- BPSK synchronous DS-CDMA system that employs Gold sequences of length $N = 31$.
- Normalized step-size CMV-SG and CCM-SG
  RLS-like versions: CMV-RLS and CCM-RLS
  Proposed: CMV-AP and CCM-AP, $P = 2, 3$.
- Channel estimation: SG algorithm from Doukopoulos and Moustakides.

Conclusions

This work presented low complexity blind constrained data-reusing adaptive filtering algorithms based on the minimum variance and constant modulus cost functions. The proposed techniques were evaluated through computer simulations and comparisons with existing SG and RLS implementations in a CDMA interference suppression scenario were performed. The proposed algorithms have shown performance close to the RLS implementations at significantly lower complexity.

Experiment I: BER (bit error rate) performance under fading ($f_{\text{d}}T = 10^{-4}$) in non-stationary scenario. The system starts with $K = 8$ users whose power distribution follows a log-normal random variable with standard deviation (sd) equal to 1.5 dB. At 1000 symbols, 4 users enter the cell and the power control is loosened, resulting in a power distribution with sd equal to 3 dB for all users.



Experiment II: SINR (signal-to-interference-plus noise ratio) performance in a 12-user, moderate near-far scenario and under faster ($f_{\text{d}}T = 10^{-3}$) fading. We assume that the user of interest is User 1. One interferer has a power level 10 dB above and another has 7 dB above the desired user. The remaining 9 interferers have the same power as the desired user, which corresponds to $E_{\text{b}}/N_0 = 15$ dB.