

# PERFORMANCE COMPARISON OF MINIMUM VARIANCE SINGLE CARRIER AND MULTICARRIER CDMA RECEIVERS

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## Abstract

This work provides comparisons between CDMA-based multiple access systems in a single and multicarrier fashion. Both zero padding and cyclic prefix types of guard intervals are considered. Comparisons include different performance measures such as signal-to-interference plus noise ratio (SINR), bit error rate (BER), and robustness against channel order overestimation. We also address the effects of finite number of samples when estimating the detector filter. In order to allow a fair comparison, blind detection based on the minimum variance is assumed for all considered systems. It is shown through computer simulations that multicarrier CDMA performs better than single carrier CDMA and is more robust against channel order overestimation.

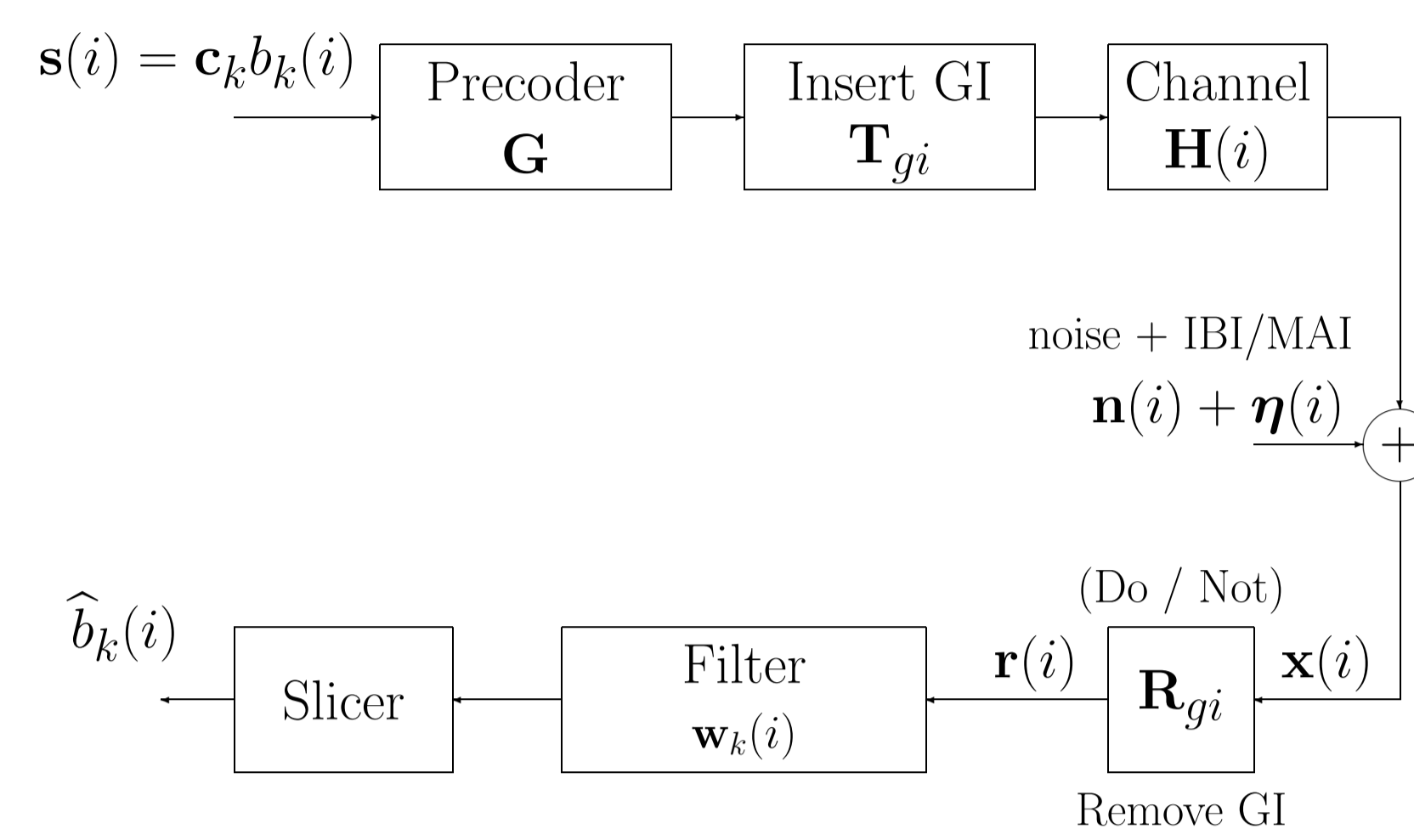
## Block Transmission Model

**Goal:** Transmit  $m$  chips of a spreaded symbol,  $\mathbf{s}_k(i) = \mathbf{c}_k b_k(i)$ , in blocks, allowing the suppression of the interblock interference, in a SC or MC fashion.

- $\mathbf{r}(i)$  the observation vector of dimension  $P \times 1$  for ZP or  $M \times 1$  for CP;

$$\mathbf{r}(i) = \mathbf{R}_{gi} \mathbf{H}(i) \mathbf{T}_{gi} \mathbf{G} \mathbf{s}(i) + \mathbf{R}_{gi} [\boldsymbol{\eta}(i) + \mathbf{n}(i)]$$

- $\mathbf{H}(i)$  a  $P \times P$  channel convolution Toeplitz matrix;



Transmission System	$\mathbf{s}$	$\mathbf{G}$	$\mathbf{T}_{gi}$	$\mathbf{R}_{gi}$
SC-CDMA-CP	$b_k \mathbf{c}_k$	$\mathbf{I}_M$	$\mathbf{T}_{cp}$	$\mathbf{R}_{cp}$
SC-CDMA-ZP	$b_k \mathbf{c}_k$	$\mathbf{I}_M$	$\mathbf{T}_{zp}$	$\mathbf{R}_{zp}$
MC-CDMA-CP	$b_k \mathbf{c}_k$	$\mathbf{F}^H$	$\mathbf{T}_{cp}$	$\mathbf{R}_{cp}$
MC-CDMA-ZP	$b_k \mathbf{c}_k$	$\mathbf{F}^H$	$\mathbf{T}_{zp}$	$\mathbf{R}_{zp}$

$$\mathbf{T}_{zp} = \begin{bmatrix} \mathbf{I}_M \\ \mathbf{0}_{G \times M} \end{bmatrix} \quad \mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{G \times M-G} & \mathbf{I}_G \\ & \mathbf{I}_M \end{bmatrix}$$

$$\mathbf{R}_{zp} = \mathbf{I}_P \quad \mathbf{R}_{cp} = \begin{bmatrix} \mathbf{0}_{M \times G} & \mathbf{I}_M \end{bmatrix}$$

$$\mathbf{F} = M\text{-point FFT, } \mathbf{F}\mathbf{F}^H = \mathbf{F}^H\mathbf{F} = \mathbf{I}_M$$

## Equivalent Model:

- Observation vector can be written as:

$$\mathbf{r}(i) = \sum_{k=1}^K \mathbf{C}_k \mathbf{h}_k(i) b_k(i) + \mathbf{n}(i)$$

$\mathbf{C}_k$  is a code related matrix for user  $k$  and  $b_k$  its transmitted symbol. and  $\mathbf{h}_k(i)$  is the impulse response of the channel for the user  $k$ .

## Minimum Variance Receivers

For the MV criterion, the cost function is

$$J_{MV} = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \mathbf{C}_k^H \mathbf{w}_k = \mathbf{g}$$

By the Lagrange multipliers method, for a given  $\mathbf{g}$ :

$$\mathbf{w}_{k,o} = \mathbf{R}^{-1} \mathbf{C}_k (\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k)^{-1} \mathbf{g}$$

Maximize  $J_{MV}(\mathbf{w}_{k,o})$  to obtain  $\mathbf{g}$

$$\mathbf{g}_o = \arg \max_{\|\mathbf{g}\|=1} \mathbf{g}^H (\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k)^{-1} \mathbf{g} = \arg \max_{\|\mathbf{g}\|=1} \mathbf{g}^H \mathbf{A} \mathbf{g}$$

## Perturbation Analysis

Realistic situations: Do not have exact  $\mathbf{R}$ .

$$\widehat{\mathbf{R}}(i) \Rightarrow \delta \mathbf{R}(i) \Rightarrow \delta \mathbf{g}_o \Rightarrow \delta \mathbf{w}_o(i)$$

In terms of  $\delta \mathbf{R}(i)$ , the SINR is written as:

$$\widehat{\text{SINR}}_k(i) = \frac{\|\mathbf{w}_o^H \mathbf{c}_k\|^2 + f_1(\delta \mathbf{R}(i))}{\mathbf{w}_o^H \mathbf{R}_I \mathbf{w}_o + f_2(\delta \mathbf{R}(i))}$$

$$f_1(\delta \mathbf{R}(i)) = \mathbf{w}_o^H \mathbf{E} \left[ \delta \mathbf{R}(i) \mathbf{A}_k^H \mathbf{c}_k \mathbf{c}_k^H \mathbf{A}_k \delta \mathbf{R}(i) \right] \mathbf{w}_o$$

$$f_2(\delta \mathbf{R}(i)) = \mathbf{w}_o^H \mathbf{E} \left[ \delta \mathbf{R}(i) \mathbf{A}_k^H \mathbf{R}_I \mathbf{A}_k \delta \mathbf{R}(i) \right] \mathbf{w}_o$$

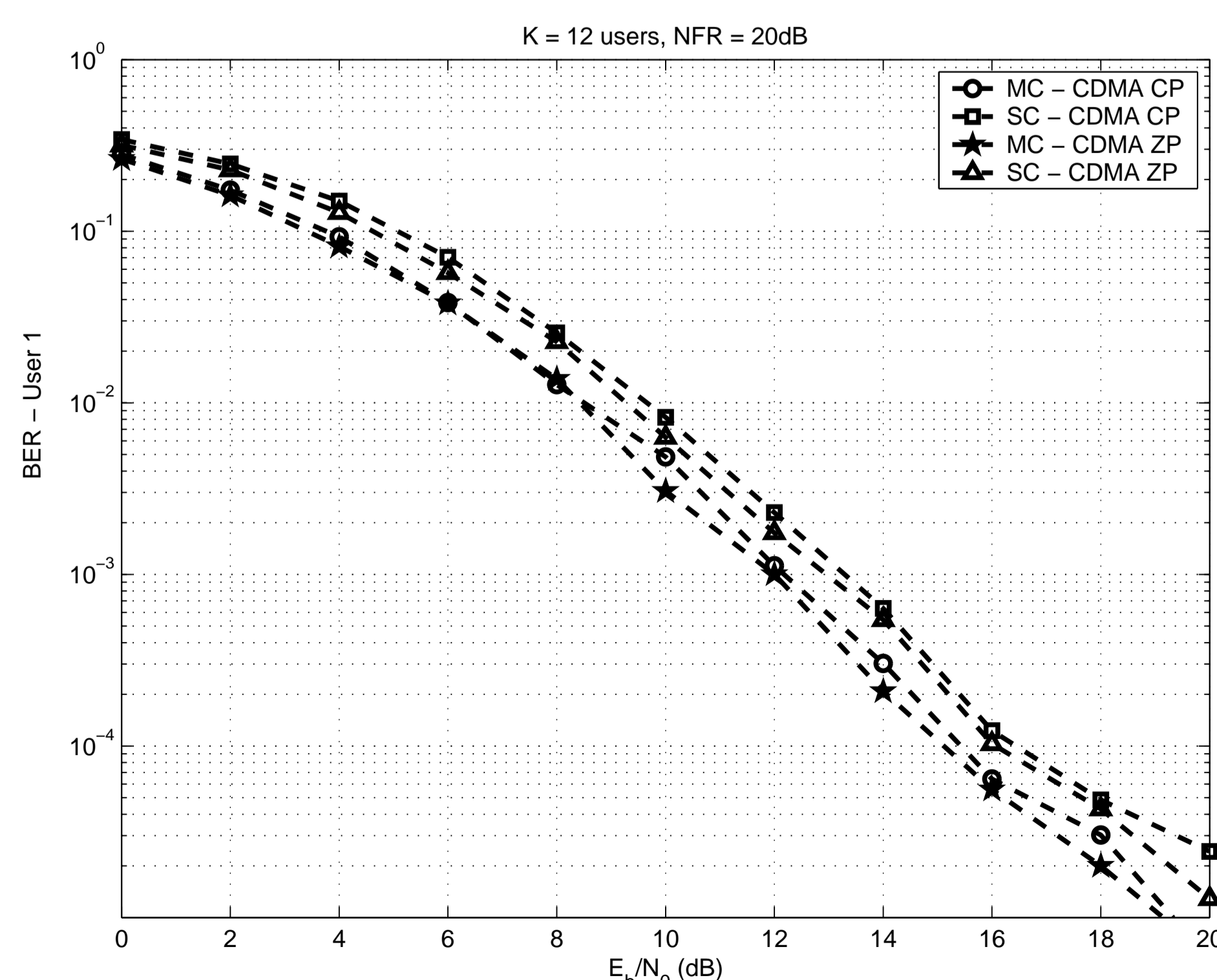
$$\mathbf{A}_k = \left[ \mathbf{R}^{-1} \mathbf{C}_k (\mathbf{A} - \lambda_{\mathbf{A}, \max} \mathbf{I})^{-1} \mathbf{C}_k^H - \mathbf{I} \right] \mathbf{R}^{-1}$$

$\mathbf{R}_I$  = interference + noise correlation matrix.

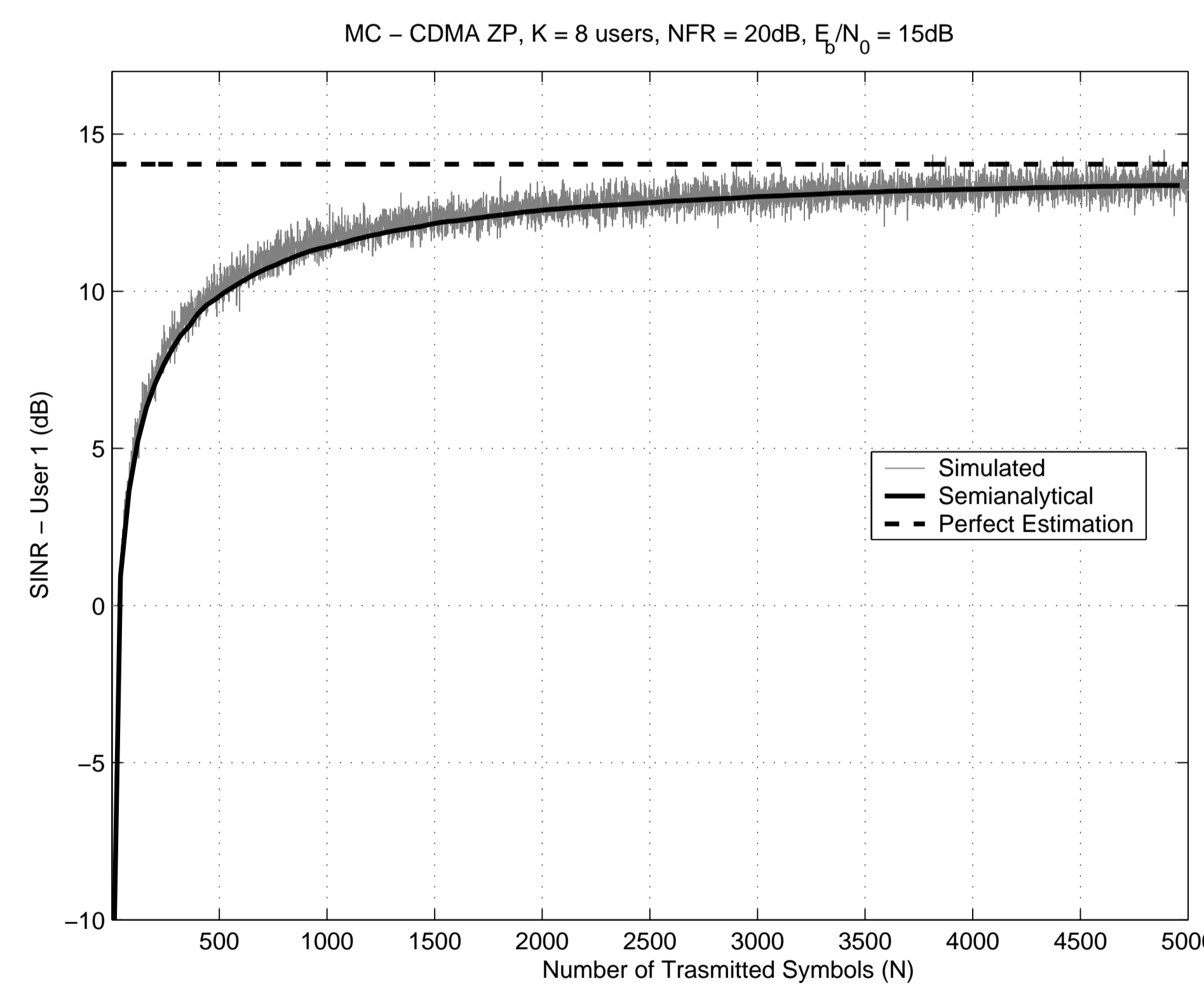
## Simulations and Results

- BPSK synchronous DS-CDMA. Gold sequences of length  $N = 31$ .  $NFR = 20dB$
- Channel length  $L = 4$
- GI length =  $L - 1$  (unless stated otherwise)

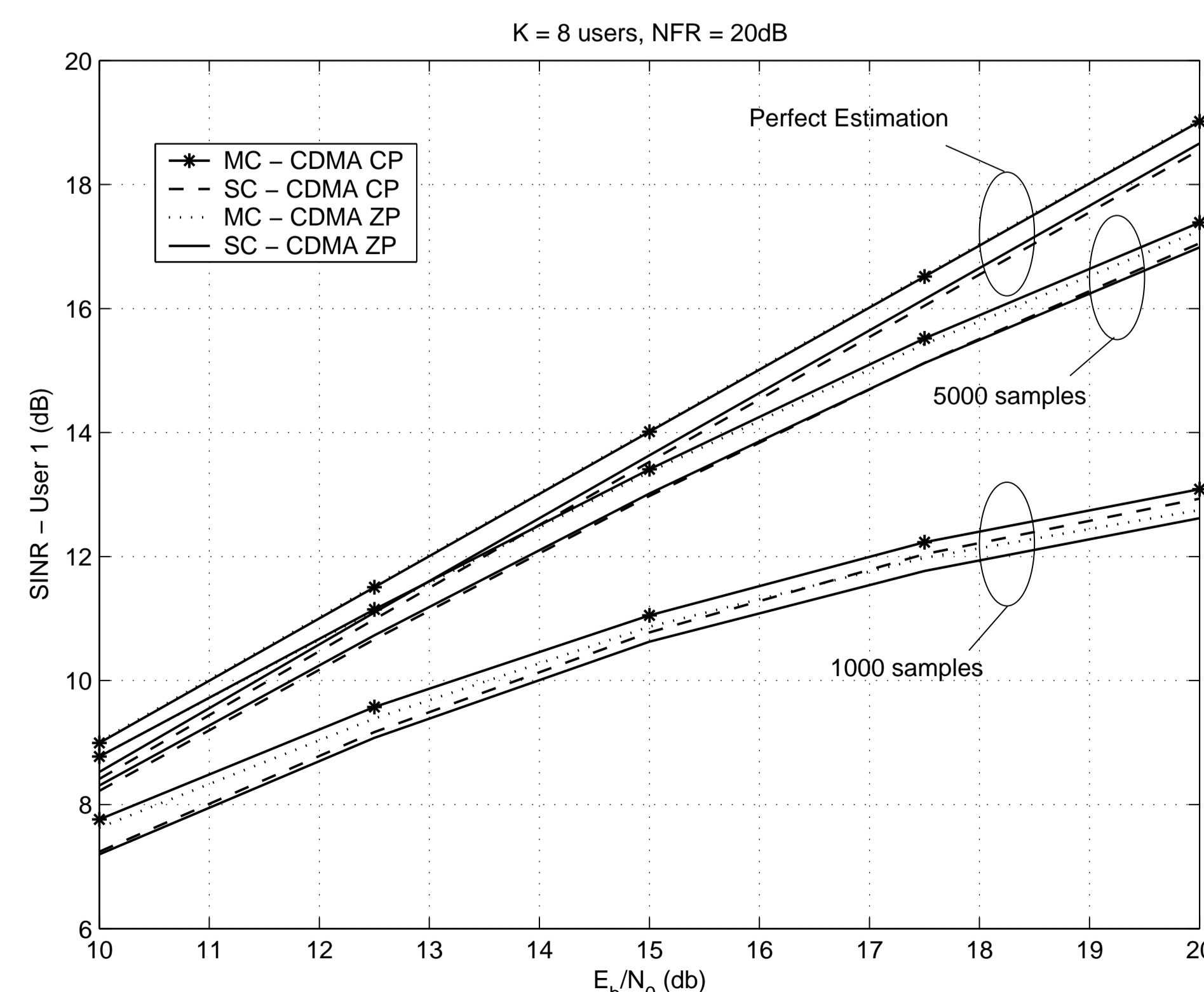
**Experiment I:** BER performance vs  $E_b/N_0$ .  $K = 12$  users. Channel randomly drawn from zero-mean complex Gaussian random variable, kept fixed throughout the experiment,  $\|h\|^2 = 1$ .



**Experiment II:** Semianalytical SINR vs number of transmitted symbols.  $K = 8$  users, the desired user has a  $E_b/N_0 = 15dB$ . Channel model same as I.

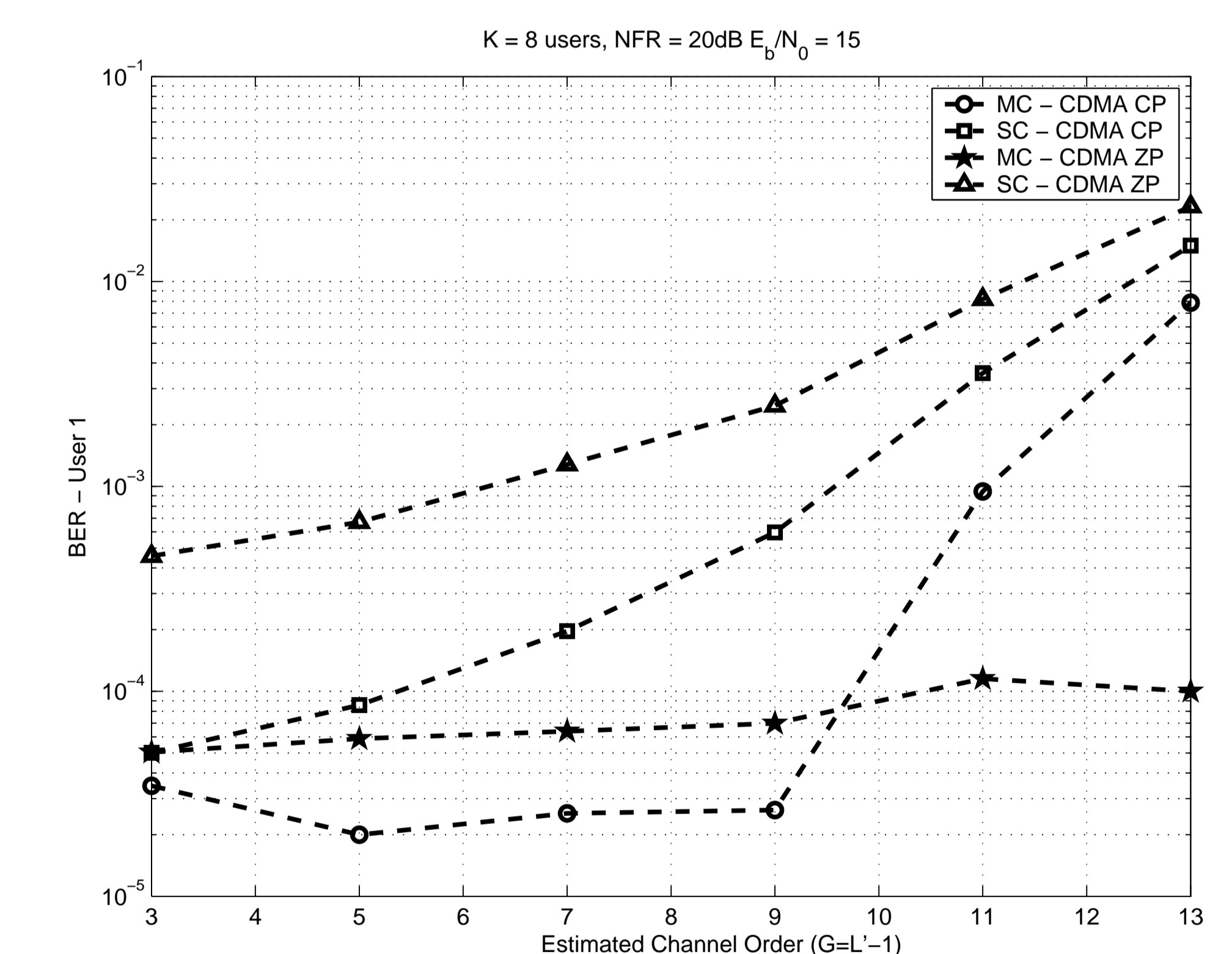


**Experiment III:** Semianalytical SINR vs  $E_b/N_0$ .  $K = 8$  users.



**Experiment IV:** BER vs estimated channel order.  $K = 8$  users. Channel estimation procedure uses the

given length  $G$  of the GI as the unknown channel order, that is,  $G = L' - 1$ . Fixed channel.



## Conclusions

In this paper we have compared single and multicarrier block transmission CDMA-based multiple access systems. The comparison was carried out in several performance measures with a minimum variance receiver. The effect of finite-data-samples estimation was also considered. Under the test conditions, it is shown that in terms of BER and SINR, MC-CDMA-ZP performs slightly better than the other systems, MC-CDMA-CP, SC-CDMA-CP, and SC-CDMA-ZP. Also, we concluded that the CMV receiver for multicarrier transmission was less sensitive to channel order overestimation than their single carrier counterparts.