Reduced Complexity Blind Channel Estimation for Adaptive Constrained Minimum Variance Receivers in MC-CDMA Systems

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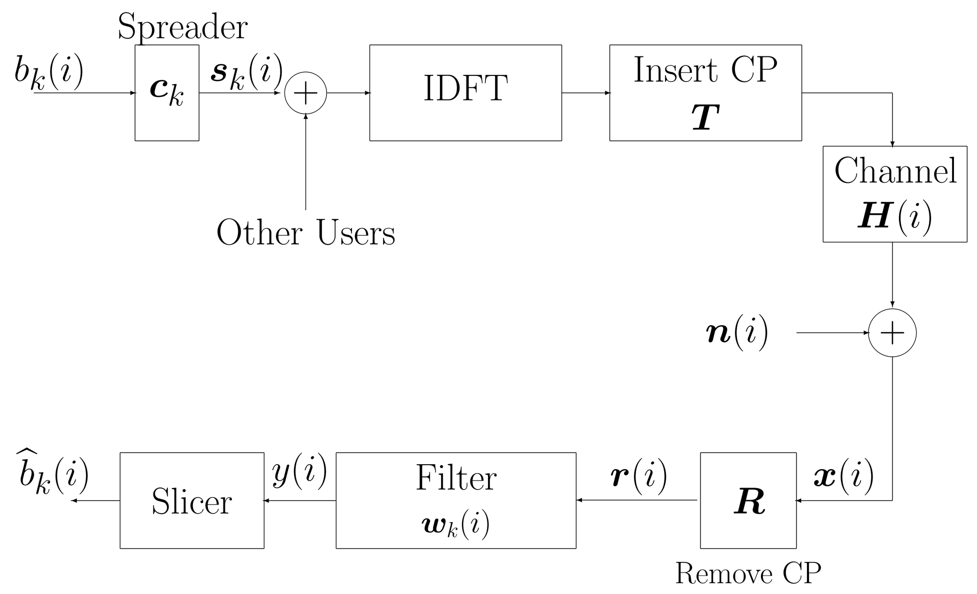
Abstract

In this paper we propose a blind adaptive channel estimator for multicarrier CDMA systems. The proposed algorithm is derived from the stochastic gradient solution for the linearly constrained minimum variance detector and it is shown to present lower computational complexity when compared to a well-known solution. It is shown through computer simulations that both the channel estimation and receiver filter achieve a slightly better performance than previously proposed methods.

System Model

Goal: Multicarrier modulation of the M chips from a spreaded symbol, $\mathbf{s}_k(i) = \mathbf{c}_k b_k(i)$.



Observation vector:

$$\boldsymbol{r}(i) = \boldsymbol{R}\boldsymbol{H}(i)\boldsymbol{T}\boldsymbol{W}^H \sum_{k=1}^{K} \boldsymbol{c}_k b_k(i) + \boldsymbol{R}\boldsymbol{n}(i)$$

- Code $\boldsymbol{c}_k$ of M chips per symbol;
- $\boldsymbol{W}^H \leftarrow$ multicarrier modulation (IDFT);
- Guard interval (length G): Cyclic prefix
- $\boldsymbol{H}(i) \leftarrow$ multipath channel (length L).

Equivalent Model:

- Obsevation vector can be written as:

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} \boldsymbol{C}_k \boldsymbol{h}(i) b_k(i) + \boldsymbol{n}'(i)$$

$\boldsymbol{C}_k$ is an $M \times L$ code related circulant matrix for user k, containing circularly-shifted versions of the k-th user transformed spreading sequence, $\boldsymbol{W}^H\boldsymbol{c}_k$.

Constrained Minimum Variance Receivers

For the MV criterion, the cost function is

$$J_{MV} = \boldsymbol{w}_k^H \boldsymbol{R}_{\boldsymbol{rr}} \boldsymbol{w}_k \quad \text{s.t.} \quad \boldsymbol{C}_k^H \boldsymbol{w}_k = \boldsymbol{g}$$

By the Lagrange multipliers method, for a given $\mathbf{g}$:

$$\boldsymbol{w}_{k,o} = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}\boldsymbol{C}_k(\boldsymbol{C}_k^H \boldsymbol{R}_{\boldsymbol{rr}}^{-1}\boldsymbol{C}_k)^{-1}\boldsymbol{g}$$

Maximize $J_{MV}(\boldsymbol{w}_{k,o})$ to obtain $\boldsymbol{g}$

$$\boldsymbol{g}_o = \arg\max_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H(\boldsymbol{C}_k^H \boldsymbol{R}_{\boldsymbol{rr}}^{-1}\mathbf{C}_k)^{-1}\boldsymbol{g}.$$

Blind Adaptive SG Algorithm

Considering the Lagrangian of the cost function, Xu and Tsatsanis update the both vectors as

$$\boldsymbol{w}_k(i+1) = \boldsymbol{w}_k(i) - \mu_{\boldsymbol{w}} \nabla_{\boldsymbol{w}_k} \mathcal{L}_{MV}$$
$$\boldsymbol{g}(i+1) = \boldsymbol{g}(i) + \mu_{\boldsymbol{g}} \nabla_{\boldsymbol{g}} \mathcal{L}_{MV}$$

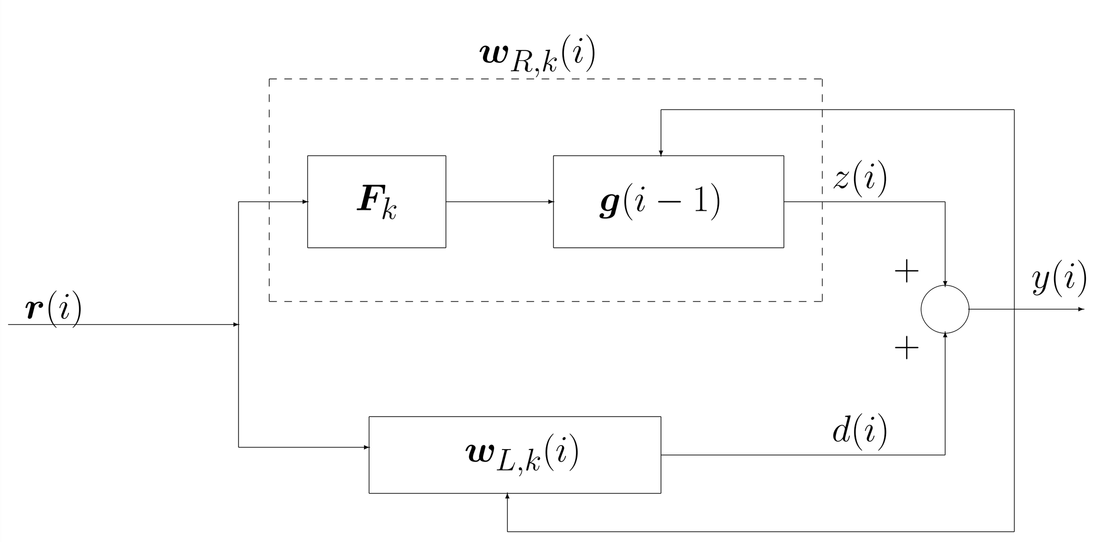
Enforcing the constraints and by using an instantaneous approximation $\hat{\boldsymbol{R}}_{\boldsymbol{rr}}(i) = \boldsymbol{r}(i)\boldsymbol{r}^H(i)$ for $\boldsymbol{R}_{\boldsymbol{rr}}$:

$$\boldsymbol{w}_k(i+1) = \boldsymbol{P}_k\left[\boldsymbol{w}_k(i) - \mu_w y^*(i)\boldsymbol{r}(i)\right] + \boldsymbol{F}_k\boldsymbol{g}(i)$$

$$\boldsymbol{g}(i+1) = \boldsymbol{g}(i) + \frac{\mu_g}{\mu_w}\left(\boldsymbol{I} - \frac{\boldsymbol{g}(i)\boldsymbol{g}^H(i)}{\boldsymbol{g}^H(i)\boldsymbol{g}(i)}\right)\left(\boldsymbol{C}_k^H\boldsymbol{C}_k\right)^{-1} \times \left[\mu_w \boldsymbol{C}_k^H y^*(i)\boldsymbol{r}(i) + \boldsymbol{g}(i) - \boldsymbol{C}_k^H\boldsymbol{w}_k(i)\right]$$

where $\boldsymbol{P}_k = \boldsymbol{I} - \boldsymbol{C}_k(\boldsymbol{C}_k^H\boldsymbol{C}_k)^{-1}\boldsymbol{C}_k^H$
$\boldsymbol{F}_k = \boldsymbol{C}_k(\boldsymbol{C}_k^H\boldsymbol{C}_k)^{-1}$, $y(i) = \boldsymbol{w}_k^H(i)\boldsymbol{r}(i)$.

New Channel Estimation Algorithm



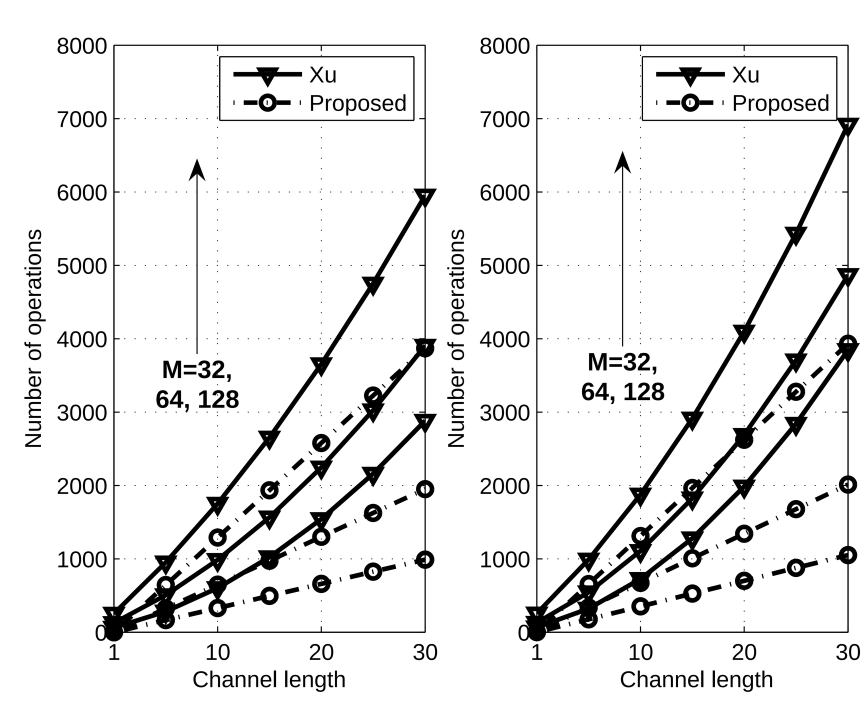
At instant $i-1$, $\boldsymbol{w}_k(i)$ is updated as:

$$\boldsymbol{w}_k(i) = \boldsymbol{w}_{L,k}(i) + \boldsymbol{w}_{R,k}(i)$$

where
$\boldsymbol{w}_{L,k}(i) = \boldsymbol{P}_k\left[\boldsymbol{w}_k(i-1) - \mu_w y^*(i-1)\boldsymbol{r}(i-1)\right]$
$\boldsymbol{w}_{R,k}(i) = \boldsymbol{F}_k\boldsymbol{g}(i-1)$
Then the cost function is $J_{MV}(i) = \boldsymbol{w}_{L,k}^H(i)\boldsymbol{R}_{\boldsymbol{rr}}\boldsymbol{w}_{L,k}(i) + \boldsymbol{w}_{R,k}^H(i)\boldsymbol{R}_{\boldsymbol{rr}}\boldsymbol{w}_{R,k}(i) + 2\Re\{\boldsymbol{w}_{R,k}^H(i)\boldsymbol{R}_{\boldsymbol{rr}}\boldsymbol{w}_{L,k}(i)\}$.
Using an instantaneous approximation $\hat{\boldsymbol{R}}_{\boldsymbol{rr}}(i) = \boldsymbol{r}(i)\boldsymbol{r}^H(i)$ for $\boldsymbol{R}_{\boldsymbol{rr}}$, then $\boldsymbol{g}(i+1)$ is updated following the stochastic gradient direction, as:

$$\boldsymbol{g}(i+1) = \boldsymbol{g}(i) + \mu_g y^*(i)\boldsymbol{F}_k^H\boldsymbol{r}(i)$$

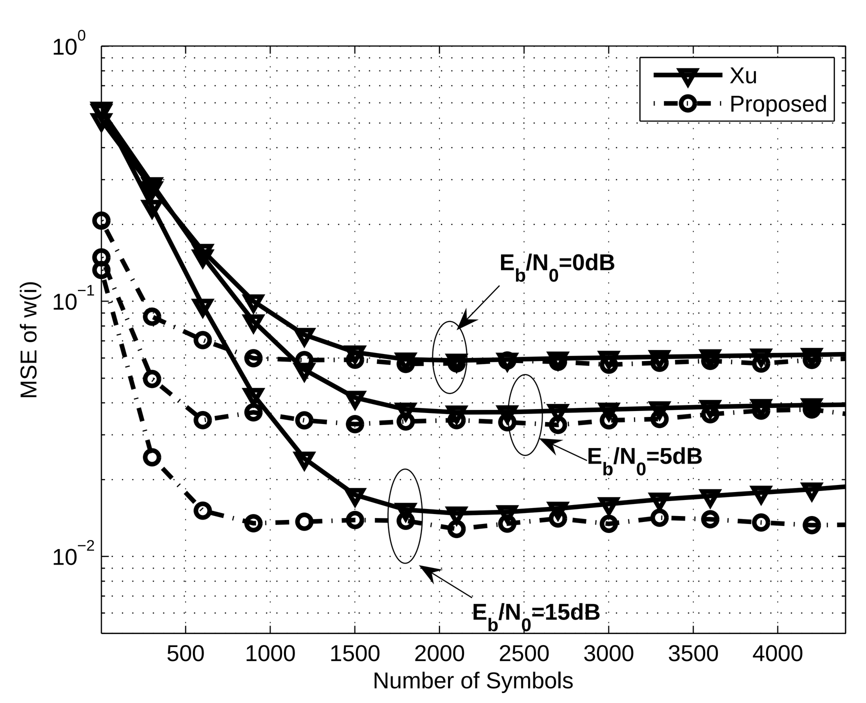
Complexity:



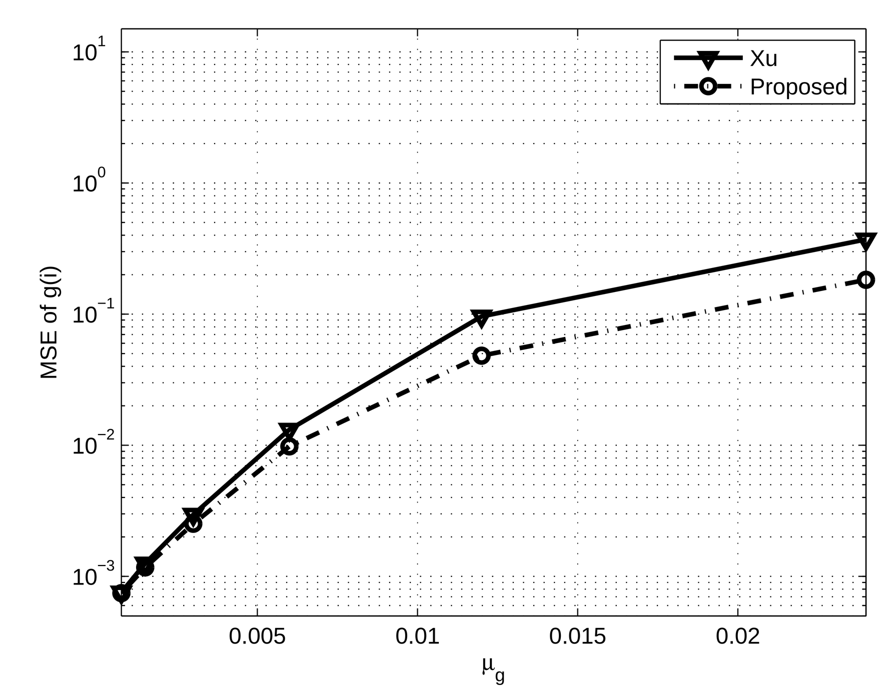
Simulations and Results

BPSK synchronous MC-CDMA. Gold sequences of length $N = 31$; channel length $L = 4$; $G = L - 1$; one interferer $\rightarrow$ same power level + two interferers $\rightarrow$ 5 dB above + two $\rightarrow$ 10 dB above; average of 1000 experiments.

Experiment I: Receiver filter mean square error performance, $\mathsf{E}\left[\|\boldsymbol{w}_{k,o} - \boldsymbol{w}_k(i)\|^2\right]$.



Experiment II: Channel mean square error performance, $\mathsf{E}\left[\|\boldsymbol{h} - \boldsymbol{g}(i)\|^2\right]$, versus the step size, $E_b/N_0 = 10$ dB.



Conclusions

In this paper we proposed a new blind channel estimation algorithm derived from the instantaneous output variance of the stochastic gradient solution for the linearly constrained adaptive algorithm. It was shown that the proposed algorithm has linear order computational complexity. Through simulations we demonstrated that with an appropriate choice of the step sizes, the proposed algorithm has a slightly better performance than well-known algorithms while reduces its computational complexity.